Waves & Sounds

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WAVES

Transmission of energy from one point of space to another due to oscillation of material particles or electric field constitute a wave motion.

TYPES OF WAVES

There are two types of waves:

ELECTROMAGNETIC RADIATION

These involve transmission of energy due to oscillation of electric field. They do not require medium to travel.

MATERIAL WAVES

These involve transmission of energy due to oscillation of material particles. They require medium to travel.

CLASSIFICATION OF WAVES BY NATURE OF DISTURBANCE PROPAGATION

TRANSVERSE WAVES

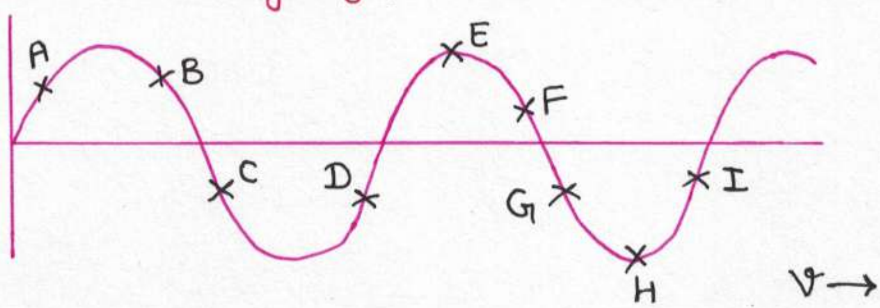
When the motion of particles is perpendicular to the directⁿ. of propogation to the direction of distance, we call it transverse waves.

LONGITUDINAL WAVES

When the motion of particles is along the direction of propagation of disturbance, we call it longitudinal waves.

Eg- Sound waves.

Que.) A transverse wave is travelling on a string from left to right as shown. What will be the direction of instantaneous velocity of particle A to I.



HINT: To see the direction of velocity of any point check the velocity of its previous point for future configuration of that point & for past configuration check the velocity of a point next to it.

Velocities of all points will be only upward or downward as its a transverse wave.

$v_A = D$

$v_B = U$

$v_C = U$

$v_D = D$

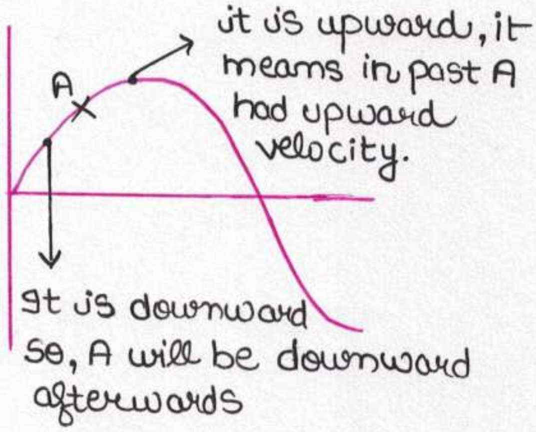
$v_E = 0$

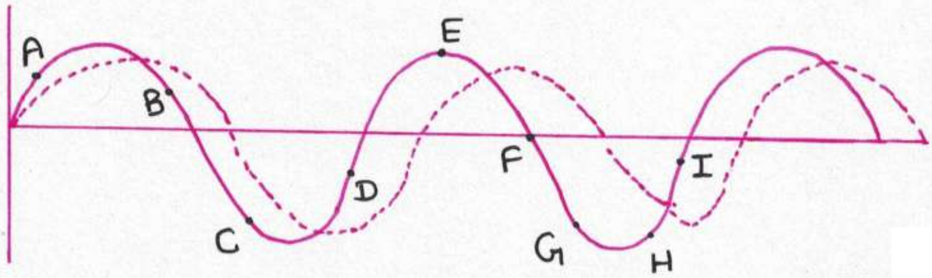
$v_F = U$

$v_G = U$

$v_H = 0$

$v_I = U$



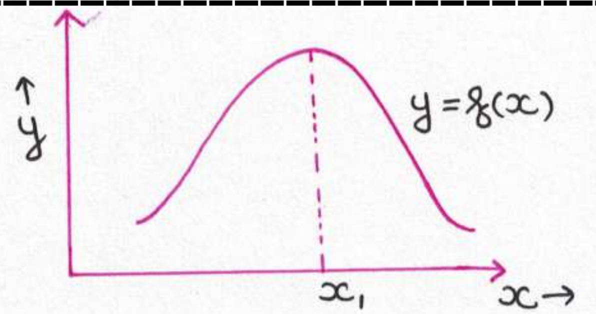


NOTE: If the direction of velocity of wave is reversed then the velocity of all points would be reversed.

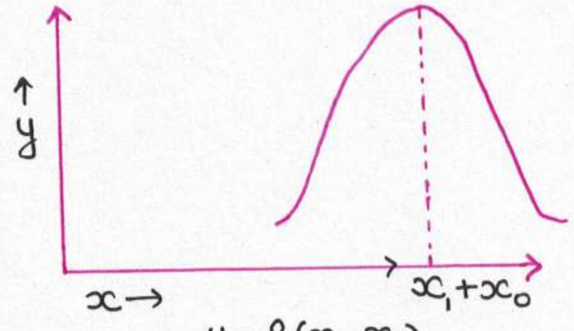
FUNCTIONS REPRESENTING A TRAVELLING WAVE

Consider three functions $y = f(x)$, $y = f(x - x_0)$ and $y = f(x + x_0)$

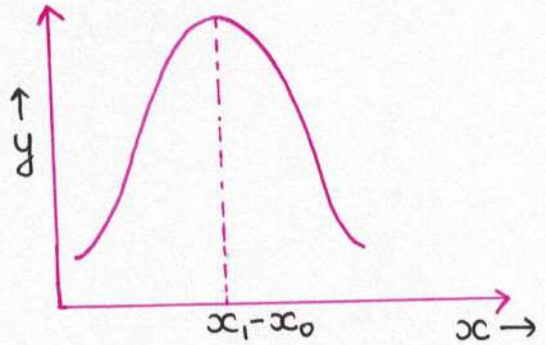
It can be easily seen that the 3 functions will have identical shapes, however the second function will be shifted to the right by x_0 units while the third function will be shifted to the left by x units.



$y = f(x)$



$y = f(x - x_0)$



$y = f(x + x_0)$

Now suppose we were to make x_0 a time dependent function $x_0 = vt$, then the second and third function will represent a graph which shifts by an amount v per unit time.

The second function will have peak shifting to right while the third function will have peak shifting to left. Thus these function can be used to represent a disturbance travelling in one direction, we say that any continuous and bounded function of the form $y = f(x \pm vt)$ represents a wave travelling along x-axis with a speed v.

Que.) For the following function find wave velocity where ever applicable. Also identify effects.

(a) $y = (4x - 3t)$

$y = 4\left(x - \frac{3}{4}t\right)$ (not bounded)

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$v = \frac{3}{4}$ in $+x$ direction

(b) $y = \sin(3x+6t)$

$y = \sin 3(x+2t)$

$v = 2$ in $-x$ direction

(c) $y = A\sin(Kx - \omega t)$

$y = A \sin K\left(x - \frac{\omega}{K}t\right)$

$v = \frac{+\omega}{K}$ in '+' x direction

(d) $y = x^2 - 4t$

Cannot be represented as a function of $f(x-vt)$. So its not a wave.

(bounded f(m) means the f should not have value of ∞ for any x)

DIFFERENTIAL FORM OF A WAVE FUNCTION

Any continuous or a bounded function which satisfies the differential equation

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

represents a wave

Verification:

$$y = f(x+vt)$$

$$\frac{\partial y}{\partial x} = f'(x+vt)$$

$$\frac{\partial^2 y}{\partial x^2} = f''(x+vt) \quad \text{— (1)}$$

$$\frac{\partial y}{\partial t} = v f'(x+vt)$$

$$\frac{\partial^2 y}{\partial t^2} = v^2 f''(x+vt) \quad \text{— (2)}$$

equating (1) and (2)

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

PHYSICAL MEANING OF PARTIAL DERIVATIVES

$\frac{\partial y}{\partial t} \rightarrow$ particle velocity

$\frac{\partial^2 y}{\partial t^2} \rightarrow$ particle acceleration

$\frac{\partial y}{\partial x} \rightarrow$ slope of the string

$\frac{\partial^2 y}{\partial x^2} \rightarrow$ Curvature of the string

Que.) A transverse wave is represented by the function $y = 10\sin\left(\frac{\pi}{6}x + \frac{\pi}{6}t\right)$. Find

(i) velocity of the particle $x=1$ at $t=1$

(ii) acceleration of the particle $x=1$ at $t=2$

(iii) amplitude of the particle $x=1$ at $t=2$

(iv) time period of particle oscillation

(v) wave velocity

(i) $$\frac{\partial y}{\partial t} = \frac{10\pi}{6}\cos\left(\frac{\pi x}{6} + \frac{\pi t}{6}\right)$$

$$= 10 \times \frac{\pi}{6} \times \frac{1}{2} \quad = \frac{5\pi}{6}\ m/s$$

(ii) $$\frac{\partial^2 y}{\partial t^2} = -10\frac{\pi^2}{36}\sin\left(\frac{\pi x}{6} + \frac{\pi t}{6}\right)$$

$$= \frac{-5\pi^2}{18}\ m/s^2$$

(iii) amplitude $= 10\,m$

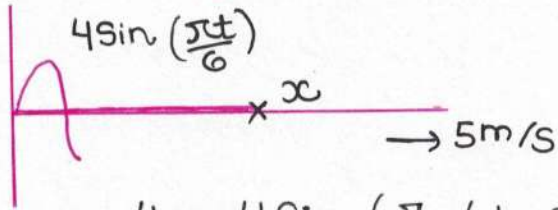
(iv) $$T = \frac{2\cancel{\pi}}{\cancel{\pi}/6} = 12\ sec.$$

(v) $$y = 10\sin\left(\frac{\pi x}{6} + \frac{\pi t}{6}\right)$$

$$y = 10\sin\frac{\pi}{6}(x+t)$$

$$v = -1\ m/s$$

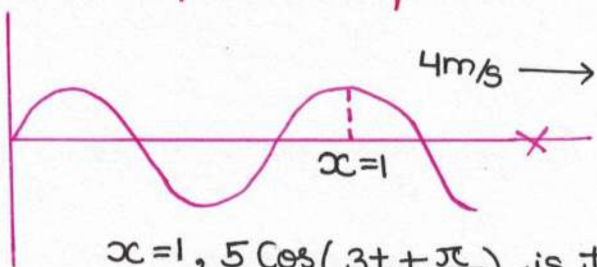
Que.) A string is tied b/w $x=0$ and $x=\infty$ and the disturbance propagated is $4\sin\left(\frac{\pi t}{6}\right)$. Find wave equation when velocity is 5 m/s (+ve x direction)



$$y = 4\sin\left(\frac{\pi}{6}\left(t-\frac{x}{5}\right)\right)$$

NOTE: $f(t-t_0)$ represents function of $f(t)$ but shifted in future. Similarly $f(t+t_0)$ represents function of $f(t)$ but shifted in past.

Que.) Repeat the previous que.

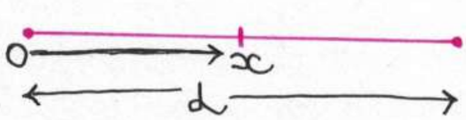


$x=1$, $5\cos\left(3t+\frac{\pi}{3}\right)$ is the disturbance

$$y = 5\cos\left(3\left(t-\frac{(x-1)}{4}\right)+\frac{\pi}{3}\right)$$

$$y = 5\cos\left(3t-\frac{3x}{4}+\frac{3}{4}+\frac{\pi}{3}\right)$$

Que.) Consider a string b/w $x=0$ & $x=L$ as shown, The incident disturbance at pt. O is given by $y_i = A\sin\omega t$. The velocity at this instant is ω/k. Find
(i) The eq^n of incident wave at general x
(ii) The incident disturbance at point P at a general time.
(iii) Suppose at point P, there is a reflection without phase change. Find the eq^n of the reflected wave assuming same velocity.
(iv) Find the resultant wave equation.



(i) $y_i = A\sin\left(\omega\left(t - \frac{Kx}{\omega}\right)\right)$

$y_i = A\sin(\omega t - Kx)$

(ii) $y_i = A\sin(\omega t - Kd)$ $\qquad (@\, x = d)$

(iii) $y_r = A\sin\left(\omega\left(t - \frac{(d-x)}{\omega/K}\right) - Kd\right)$

$y_r = A\sin(\omega t + Kx - 2Kd)$

(iv) $y_{net} = y_i + y_r$

$= 2A\sin(\omega t - Kd)\cos(-Kx + Kd)$

$= 2A\sin(\omega t - Kd)\cos(K(x-d))$

SIMPLE HARMONIC WAVE

Any wave of the form

$y = A\sin(\pm Kx \pm \omega t \pm \phi)$

$y = A\cos(\pm Kx \pm \omega t \pm \phi)$

represents a simple harmonic wave. Here, 'ω' is angular frequency of the wave. 'K' is called wave propagation const., 'A' is called amplitude of wave and ϕ is called phase const.

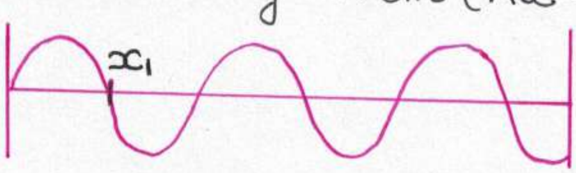
We have seen that velocity of such a wave is ω/K. The wave is travelling to the right if coefficients of x and t have opposite sign and it is travelling to the left if they have same sign.

$\pm Kx \pm \omega t \pm \phi$ is called phase of the wave and their difference is called phase difference.

TIME PERIOD OF A WAVE

The minimum time interval at which the phase of a given particle changes by 2π is called time period of the wave.

Let say the wave is $y = A\sin(Kx - \omega t + \phi)$



Phase at time t

$\Phi_1 = (Kx_1 - \omega t + \phi)$

Phase at time $(t+T)$

$\Phi_2 = (Kx_1 - \omega(t+T) + \phi)$

by definition of T

$$|\Phi_1 - \Phi_2| = 2\pi$$

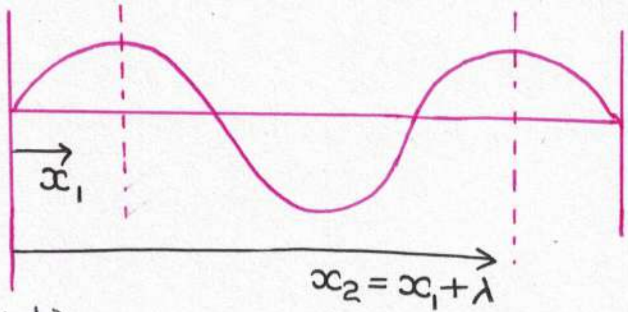
$$\omega T = 2\pi$$

$$T = \frac{2\pi}{\omega}$$

It is also the minimum time after which particle will repeat its configuration.

WAVELENGTH

Minimum separation b/w two points at a given time having a phase difference of 2π is called wavelength.



$A \sin(Kx - \omega t + \phi)$

$$\Phi_1 = Kx_1 - \omega t + \phi$$

$$\Phi_2 = Kx_2 - \omega t + \phi$$

$$|\Phi_1 - \Phi_2| = 2\pi$$

$$K|x_1 - x_2| = 2\pi$$

$$K\lambda = 2\pi$$

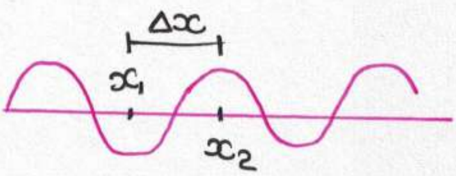
$$\lambda = \frac{2\pi}{K}$$

wavelength is also the distance by which the disturbance advances in 1 time period.

RELATION B/W PHASE DIFFERENCE & PATH DIFFEREN

$$\Delta\Phi = K\Delta x = \frac{2\pi}{\lambda} \times \Delta x$$

$$\boxed{\underbrace{\Delta\Phi}_{\text{phase difference}} = \frac{2\pi}{\lambda} \underbrace{\Delta x}_{\text{path difference}}}$$



PHASOR ADDITION OF SIMPLE HARMONIC WAVES TRAVELLING IN SAME DIRECTION

$$y_1 = A_1 \sin(kx - \omega t + \phi_1)$$
$$y_2 = A_2 \sin(kx - \omega t + \phi_2)$$
$$y_1 + y_2 = A \sin(kx - \omega t + \phi)$$

where $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_2 - \phi_1)}$

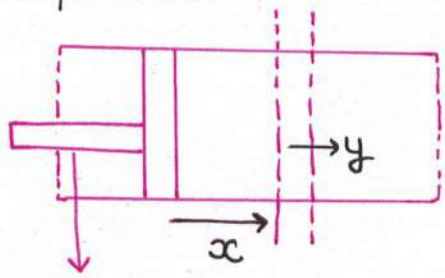
$$\tan\phi = \frac{A_1 \sin\phi_1 + A_2 \sin\phi_2}{A_1 \cos\phi_1 + A_2 \cos\phi_2}$$

LONGITUDINAL WAVES

Just like transverse waves, for longitudinal waves, we can write eq^n. of form:

$$y = A \sin(kx - \omega t + \phi)$$

Here x represents the coordinate of the mean position of the particle and y represents the displacement of the mean position of the particle.



CONDITION FOR CONSTRUCTIVE INTERFERENCE OF SOUND

For constructive interference, the phase difference b/w two sounds must be $2n\pi$

This corresponds to the path difference of $n\lambda$.

$$\Delta \Phi = 2n\pi$$

$$\frac{\cancel{2\pi}}{\lambda} \cdot \Delta x = \cancel{2}n\cancel{\pi}$$

$$\boxed{\Delta x = n\lambda}$$

CONDITION FOR DESTRUCTIVE INTERFERENCE OF SOUND

The phase difference should be odd multiple of π. This corresponds to path difference of odd multiple of $\lambda/2$.

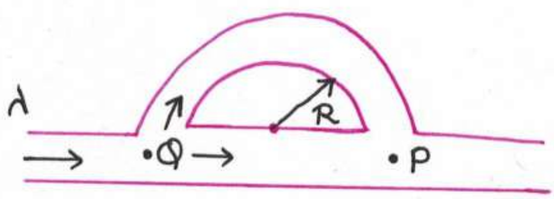
$$\Delta\Phi = (2n+1)\pi$$

$$\frac{2\pi}{\lambda}\Delta x = (2n+1)\pi$$

$$\boxed{\Delta x = (2n+1)\frac{\lambda}{2}}$$

Que:) Sound enters a branched pipe to bifurcate at point Q along two paths to recombine at point P. Find the values of R so that

(i) very loud sound is produced at point P.
(ii) There is silence at point P.
(iii) what do you think will happen at, if a mixture of several wavelengths is sent at Q.
(iv) The minimum value of R for (i) & (ii)



(i) $\pi R - 2R = n\lambda$

$$R = \frac{n\lambda}{(\pi-2)}$$

(ii) $\pi R - 2R = (2n+1)\frac{\lambda}{2}$

$$R = \frac{(2n+1)}{(\pi-2)}\frac{\lambda}{2}$$

(iii) For constructive interference,

$$\lambda = \frac{(\pi-2)R}{n}$$

(iv) R_{min} for (i) $R = \frac{\lambda}{\pi-2}$

R_{min} for (ii) $R = \frac{\lambda}{(\pi-2)2}$

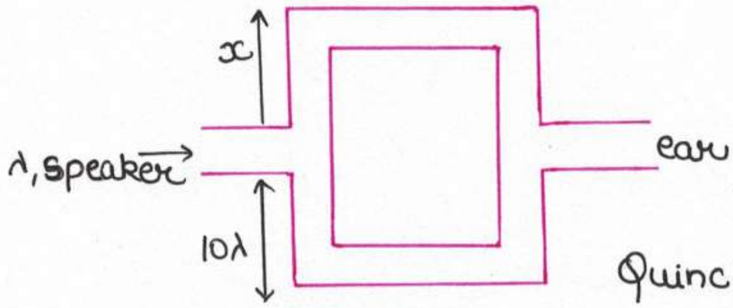
Those frequencies which will be given by the above eqn. will be boosted to the maximum.

Similarly, for destructive interference,

$$\lambda = \frac{(\pi-2)R}{n+1/2}$$

wavelengths given by the above eqⁿ will be cut to the maxᵐ. So, the overall effect will be between them.

Que.) Find the values of x for which no sound is heard.



Quincke's tube

$$20\lambda - 2x = (2n+1)\frac{\lambda}{2}$$

$$x = \frac{20\lambda - (2m+1)\frac{\lambda}{2}}{2}$$

$$x = \frac{40\lambda - 2m\lambda - \lambda}{4} = \frac{39\lambda - 2n\lambda}{4}$$

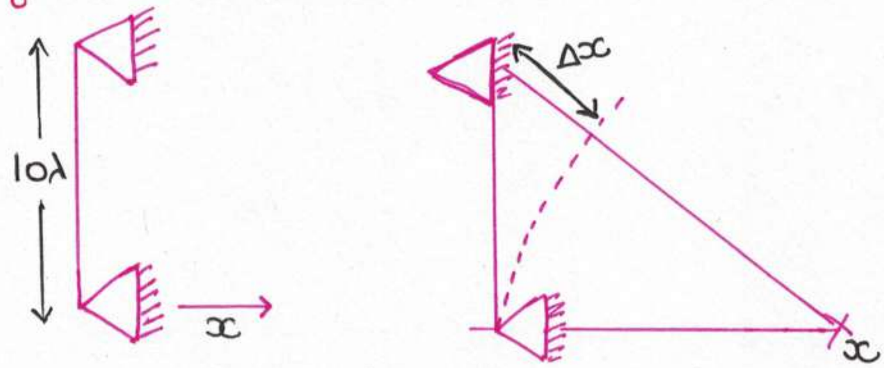
$$x = \frac{38\lambda - 2\lambda m + \lambda}{4} = \frac{(2m+1)}{4}\lambda$$

Que.) Two speakers emit monochromatic sound of wavelength λ while a peson moves along x-axis. Find

(i) Total no. of maximas

(ii) Total no. of minimas on x-axis

(iii) Also find the nearest minima



by geometry we can see that max. possible path difference is 10λ which corresponds to point O itself and minimum

possible path difference is 0 which corresponds to infinity.
If we exclude these points, we will get maximas corresponding to path difference of $9\lambda, 8\lambda, 7\lambda$ --------- 1λ (total = 9)

Similarly,

we will get minimas as

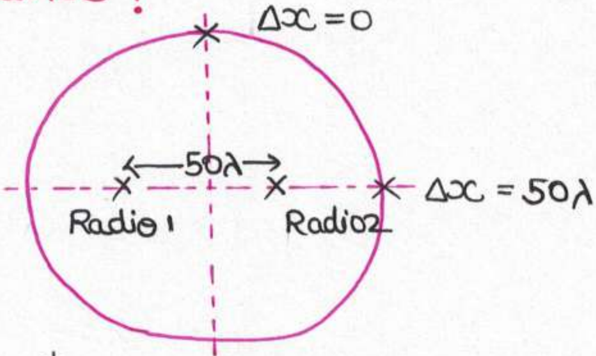
9.5λ, 8.5λ, 7.5λ, ---------- 0.5λ (total $\rightarrow$ 10)

For nearest maxima, $\Delta x = 9\lambda$

For nearest minima, $\Delta x = 9.5\lambda$

$$\sqrt{x^2 + (10\lambda)^2} - x = \frac{19}{2}\lambda$$

Que.) How many total minimas and maximas will be there along the circumference?



For 1 quadrant,

Maximas, $= 1\lambda, 2\lambda, 3\lambda$ -------- 50λ

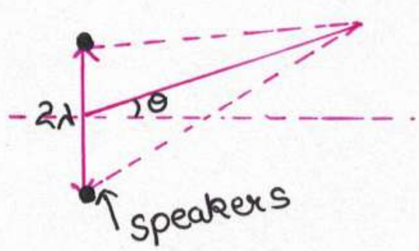
$\therefore$ Total maximas $= 49 \times 4 + 2 + 2 = 200$

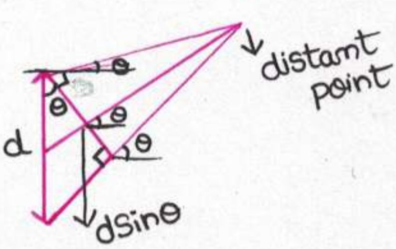
For a quadrant,

Minimas $= 0.5\lambda, 1.5\lambda,$ -------- 49.5λ

$\therefore$ Total minimas $= 50 \times 4 = 200$

Que.) Find the places where there will be strong signal & where is weak signal.

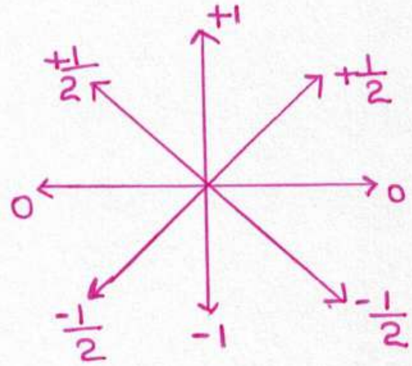




$$d\sin\theta = n\lambda$$

$$\sin\theta = \frac{n}{2}$$

$$\sin\theta = \pm\frac{1}{2}, \pm 1, 0$$



$\therefore$ 8 places will get strong signal

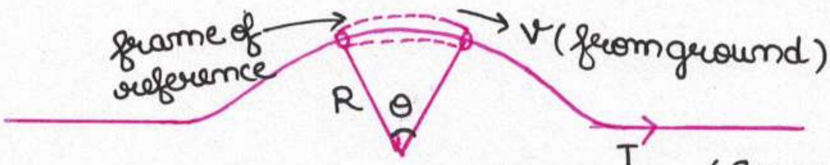
$$2\lambda\sin\theta = (2n+1)\frac{\lambda}{2}$$

$$\sin\theta = \frac{(2n+1)}{4}$$

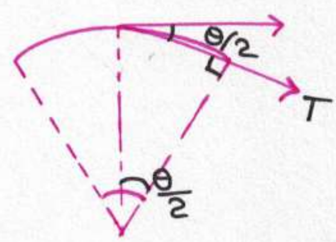
$$\sin\theta = \pm\frac{1}{4}, \pm\frac{3}{4}$$

$\therefore$ 8 places will get weak signal.

VELOCITY OF A TRANSVERSE WAVE ON A STRETCHED ELASTIC STRING



(from frame of reference i.e. the hump, $-v$ velocity is the velocity of string)



μ = mass / unit length

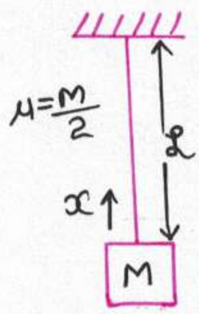
$$2T\sin\left(\frac{\theta}{2}\right) = \underbrace{\mu R\theta}_{\text{mass}} \underbrace{\frac{v^2}{R}}_{\text{centripetal acc}^n}$$

As θ is very small,

$$T\theta = \mu R\theta\frac{v^2}{R}$$

$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$

Que.) The block is given a small disturbance. Find the time after which it will reach the top.



$$T = Mg + x\mu g$$

$$v = \sqrt{\frac{\cancel{M}g\left(1+\frac{x}{L}\right)}{\cancel{M}/L}}$$

$$v = \sqrt{g(L+x)}$$

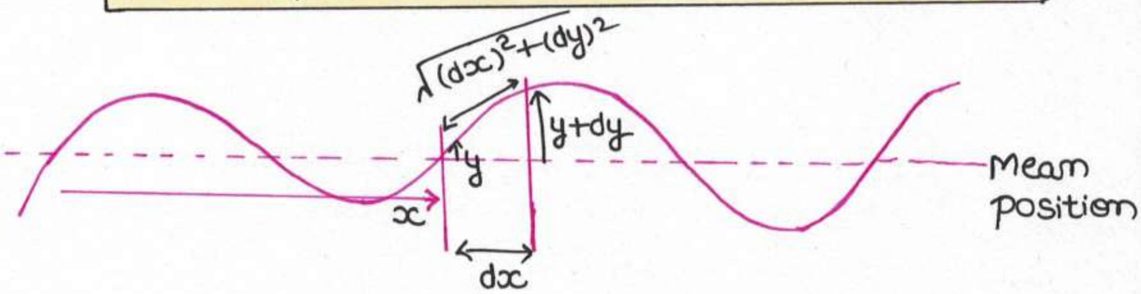
$$\int_0^L \frac{dx}{\sqrt{g(L+x)}} = \int_0^t dt$$

$$T = \left[\frac{2\sqrt{g(d+x)}}{g}\right]_0^{d}$$

$$T = \frac{2}{\sqrt{g}}\left[\sqrt{2d} - \sqrt{d}\right]$$

$$= 2(\sqrt{2}-1)\sqrt{\frac{d}{2}}$$

ENERGY, POWER & INTENSITY OF A WAVE



$\frac{\partial y}{\partial t}$ is the particle velocity

$$dK_E = \frac{1}{2}\mu\, dx\left(\frac{\partial y}{\partial t}\right)^2$$

$$dP_E = T\left(\sqrt{(dx)^2+(dy)^2} - dx\right)$$

$$dP_E = T\,dx\left(\left[1+\left(\frac{\partial y}{\partial x}\right)^2\right]^{1/2} - 1\right)$$

For a wave with small slopes,

If $\frac{\partial y}{\partial x} << 1$

$$dP_E = T\,dx\left[\cancel{1} + \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2 - \cancel{1}\right]$$

$$\underline{dP_E} = \frac{1}{2}T\,dx\left(\frac{\partial y}{\partial x}\right)^2$$

$$dP_E = \frac{1}{2}\mu v^2\left(\frac{\partial y}{\partial x}\right)^2 dx$$

$$dE = dKE + dPE$$

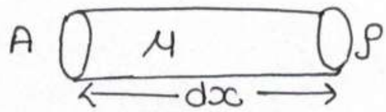
$$dE = \frac{1}{2}\mu\, dx\,\underbrace{\left[\left(\frac{\partial y}{\partial t}\right)^2 + v^2\left(\frac{\partial y}{\partial x}\right)^2\right]}_{G}$$

ENERGY PER UNIT LENGTH

$$\underset{\text{length}}{(\text{energy})}\ e_\ell = \frac{dE}{dx} = \frac{1}{2}\mu G$$

ENERGY DENSITY

$$\text{energy density} = \frac{e_\ell dx}{A dx} \quad \left(\begin{array}{c}\text{energy per}\\ \text{unit volume}\end{array}\right)$$



$$\rho A dx = \mu dx \quad (\text{mass of element})$$

$$\rho = \frac{\mu}{A}$$

$$E.D. = \frac{e_\ell . dx}{A dx}$$

$$E.D. = \frac{1}{2}\frac{\mu}{A}G = \frac{1}{2}\rho G$$

$$\boxed{e_\rho = \frac{1}{2}\rho G}$$

POWER OF A WAVE

Energy crossing the cross-section of wave per unit time is called power of a wave.

energy crossing a section in time dt

$$dE = e_\ell\, v.dt$$

$$P = \frac{dE}{dt} = \frac{1}{2}\mu v G$$

$$\boxed{P = \frac{1}{2}\mu v G}$$

INTENSITY OF A WAVE

Power crossing per unit cross-sectional area is called intensity of a wave.

Mathematically,

$$I = \frac{P}{A}$$

$$I = \frac{\frac{1}{2} \mu v G}{A}$$

$$\boxed{I = \frac{1}{2} \rho v G}$$

Que.) Show that average value of function $\cos^2\theta$ or $\sin^2\theta$ is $\frac{1}{2}$.

$$f_{avg} = \frac{\int_0^T f(v).dv}{\int_0^T d\theta} \quad (T \rightarrow T \text{ is periodicity})$$

(avg. value of a function)

$T \rightarrow$ minimum value of T such that

$$f(x+T) = f(x)$$

$$\int \cos^2\theta = \int_0^{\pi} \left(\frac{1+\cos 2\theta}{2}\right) . d\theta$$

$$= \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{1}{2} . \pi$$

$$\frac{\int \cos^2\theta}{\int_0^{\pi} d\theta} = \frac{\cancel{\pi}/2}{\cancel{\pi}} = \frac{1}{2}$$

NOTE : Average of a sum is equal to the sum of the average.

$$\left\langle \frac{1}{2} + \frac{\cos 2\theta}{2} \right\rangle$$

$$= \left\langle \frac{1}{2} \right\rangle + \left\langle \frac{\cos 2\theta}{2} \right\rangle$$

$$= \frac{1}{2} + 0$$

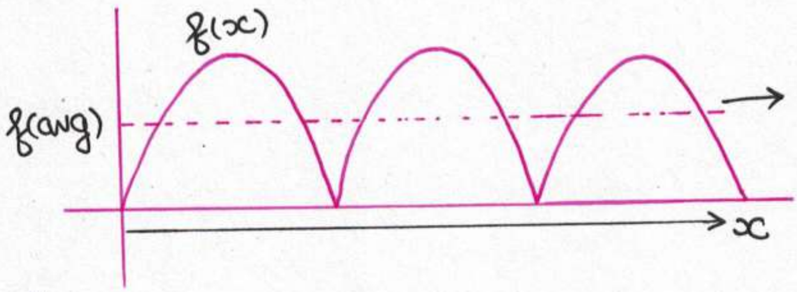
Or

$$\cos^2\theta + \sin^2\theta = 1$$

$$2 \text{ Avg.} (\cos^2\theta) = 1$$

$$\text{Avg.} (\cos^2\theta) = \frac{1}{2}$$

GRAPHICAL MEANING OF AVERAGE OF A FUNCTION



we have to make a line such that area below the line is equal to the area under the function.

$$\int_0^x f_{avg.}\,dx = \int_0^x f(x).dx$$

$$f_{avg.} = \frac{\int_0^x f(x)\,dx}{\int_0^x dx}$$

FREQUENCY

The no. of times a particle executes oscillation cycles in unit time is called its frequency.

Mathematically,

$$f = \frac{1}{T}$$

$$\omega = 2\pi f$$

ENERGY, POWER AND INTENSITY OF A SINUSOIDAL WAVE

Sin & Cos function are together known as Sinusoidal function.

$$y = A\sin(Kx - \omega t)$$

$$G = \left(\frac{\partial y}{\partial t}\right)^2 + v^2\left(\frac{\partial y}{\partial x}\right)^2$$

$$\frac{\partial y}{\partial t} = -A\omega\cos(Kx - \omega t)$$

$$\frac{\partial y}{\partial x} = AK\cos(Kx - \omega t)$$

$$G = A^2\cos^2(Kx - \omega t)\left[\omega^2 + v^2K^2\right]$$

$$G = 2A^2\omega^2\cos^2(Kx - \omega t) \qquad \left(v = \frac{\omega}{K}\right)$$

$$\langle G\rangle = A^2\omega^2$$

AVERAGE & INSTANTANEOUS POWER OF A SINUSOIDAL WAVE

$P = \frac{1}{2}\mu v G$

$P_{inst} = \mu v A^2 \omega^2 \cos^2(Kx - \omega t)$

$P_{avg} = \frac{1}{2}\mu v A^2 \omega^2$

$I = \frac{P}{A}$

$I_{inst} = \rho v A^2 \omega^2 \cos^2(Kx - \omega t)$

$I_{avg} = \frac{1}{2}\rho v A^2 \omega^2$

$I_{avg} = 2\pi^2 \rho v A^2 f^2$ $\quad (\omega = 2\pi f)$

★ Intensity is directly proportional to A^2 and f^2.

NOTE: For the interference of two waves whose frequency is same, average Intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

$(\because I \propto A^2)$

ϕ is phase diff.

Que.) In a certain infinite string, the avg. power supplied by the oscillator is P. How much power is required if a wire of double thickness (radius), double tension were used with a doubled amplitude & doubled frequency.

$$\frac{P_2}{P_1} = \frac{\frac{1}{2}(4\mu)\left(\frac{1}{\sqrt{2}}v\right)(2A)^2(2\omega)^2}{\frac{1}{2}\mu v A^2 \omega^2} = 32\sqrt{2}$$

$$v' = \sqrt{\frac{2T}{4\mu}} = \frac{1}{\sqrt{2}}v$$

$$\mu' = \frac{4M}{l}$$

(as area of cross section becomes 4 times, mass becomes 4 times)

Que.) Two identical strings carry identical powers (avg.). Their tensions are in the ratio 1:2 and frequencies are in the ratio 2:1. What is the ratio of their amplitudes

$$\left(\frac{A_1}{A_2}\right)^2 = \frac{\cancel{2P}\,\cancel{\mu}\,v_2\,\omega_2^2}{\cancel{\mu}\,v_1\,\omega_1^2 \times \cancel{2P}} = \sqrt{2} \times \frac{1}{4}$$

$$\frac{A_1}{A_2} = \left(\frac{1}{2\sqrt{2}}\right)^{1/2}$$

$$A_1 : A_2 = 1 : 2^{3/4}$$

AVERAGE ENERGY PER UNIT LENGTH

$$\langle e_l \rangle = \frac{1}{2}\mu \langle G \rangle$$

$$\langle e_l \rangle = \frac{1}{2}\mu A^2\omega^2$$

Que.) A certain sinusoidal wave is given by $y = A\sin(Kx - \omega t)$. Find the energy stored between

(i) $x = 0$ to $x = 3\lambda$

(ii) $x = 0$ to $x = \frac{\lambda}{8}$ at $t = 0$

(i) $E = \frac{1}{2}\mu G \times 3\lambda$

$$E = \frac{1}{2}\mu A^2\omega^2 \times 3\lambda$$

$$E = \frac{3\lambda}{2}\mu A^2\omega^2$$

(ii) $dE = \frac{1}{2}\mu G . dx$

$$E = \int_0^{\lambda/8} \frac{1}{2}\mu\left(2A^2\omega^2\cos^2(Kx)\right)dx \qquad (t=0)$$

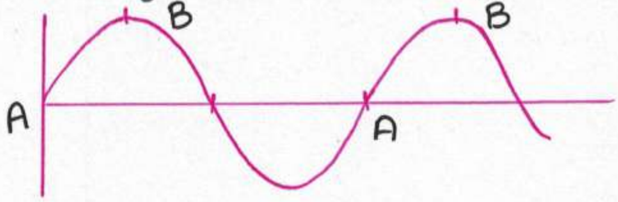
$$E = \mu A^2\omega^2 \int_0^{\lambda/8}\left(\frac{1+\cos(2Kx)}{2}\right)dx$$

$$E = \frac{1}{2}\mu A^2\omega^2\left[\frac{x + \sin(2Kx)}{2K}\right]_0^{\lambda/8}$$

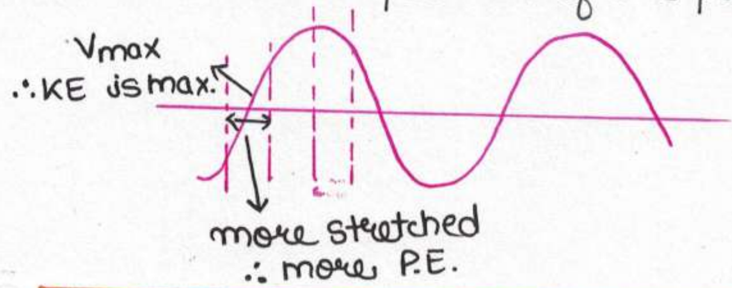
$$E = \frac{1}{2}\mu A^2\omega^2\left(\frac{\lambda}{8} + \frac{\sin\left(\frac{K\lambda}{4}\right)}{2K}\right)$$

$$E = \frac{1}{2}\mu A^2\omega^2\left(\frac{\lambda}{8} + \frac{\lambda}{4\pi}\right) \qquad \left\{K = \frac{2\pi}{\lambda}\right\}$$

NOTE: (i) Energy is independent of time because if we take integer no. of cycle we have all sections of different energies.



(ii) For a wave on the string, both the K.E. density and P.E. density is maximum near the mean position of the particle and it is minimum near the extreme position of the particle.



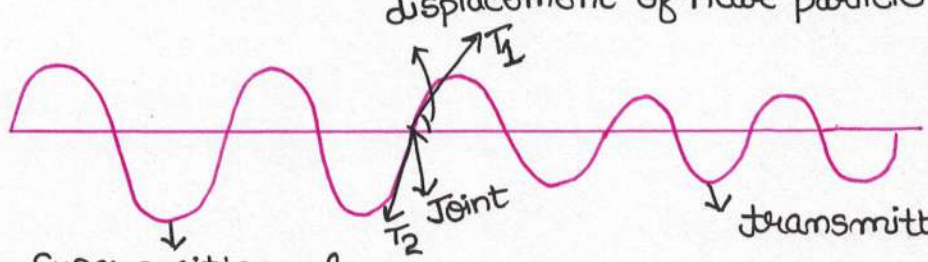
REFLECTION OF WAVES

Whenever a wave is incident upon a medium of different density, there is a transmitted wave and a reflected wave. Let y_i be the incident wave and y_r be the reflected wave and y_t be the transmitted wave, then since there is no breaking of the string, the displacement at left of the joint must be equal to the displacement at the right of the joint or we can say that at the joint,

$$y_i + y_r = y_t$$

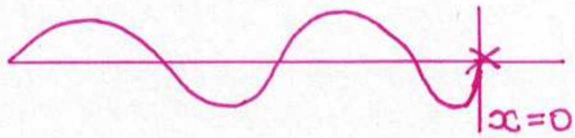
Now the joint can be considered as a massless particle. Therefore, the forces from the two sides of the joint must be equal and opposite. This can happen only if slope of left is equal to the slope of the right.

$$y_i' + y_r' = y_t'$$



as we can imagine $T_1 = T_2$ (tension)

Calculation:



Let

$$y_i = A\cos(\omega t - Kx)$$
$$y_r = A_r\cos(\omega t + Kx + \phi_r)$$
$$y_t = A_t\cos(\omega t - K'x + \phi_t)$$

K of reflected wave is same because it is in the same medium and its velocity will remain same but velocity of transmitted wave will be different. Therefore K' will be different.

ω of all the three waves is same because frequency of a wave depends on the source.

@ $x=0$, $y_i + y_r = y_t$ always

it is true that at $\omega t = 0$, $\omega t = \frac{\pi}{2}$

@ $\omega t = 0$

$$A + A_r\cos\phi_r = A_t\cos(\phi_t) \quad —(1)$$

@ $\omega t = \frac{\pi}{2}$

$$-A_r\sin\phi_r = -A_t\sin(\phi_t) \quad —(2)$$

@ $x = 0$

$$y_i' + y_r' = y_t'$$

$$AK\sin(\omega t - Kx) - A_r K\sin(\omega t + Kx + \phi_r) = A_t K'\sin(\omega t - K'x + \phi_t)$$

@ $\omega t = 0$

$$-A_r K\sin\phi_r = A_t K'\sin(\phi_t) \quad —(3)$$

@ $\omega t = \pi/2$

$$AK - A_r K\cos\phi_r = A_t K'\cos(\phi_t) \quad —(4)$$

Solving the 4 eqⁿ, we get

$$A_r = A\left(\frac{v_2 - v_1}{v_2 + v_1}\right)$$

$$A_t = A\left(\frac{2v_2}{v_2 + v_1}\right)$$

$$\phi_r = 0$$

$$\phi_t = 0$$

where $v_2 = \frac{\omega}{K'}$ and $v_1 = \frac{\omega}{K}$

→ If $v_1 > v_2$, we can say

$$A_r = \left(\frac{V_1 - V_2}{V_1 + V_2}\right)A \quad , \phi_r = \pi$$

Special Cases

1.) Medium does not change:

$v_1 = v_2$

Here we can clearly see that there will be no reflected wave.

2.) Wave going from denser medium to rarer medium:

$v_2 > v_1$

The reflected wave is in the same phase as in the incident wave.

3.) Wave going from rarer medium to denser medium:

$v_1 > v_2$

Here the reflected wave is π shifted from the incident wave.

4.) Reflection at a rigid boundary (of ∞ density):

$v_2 = 0$

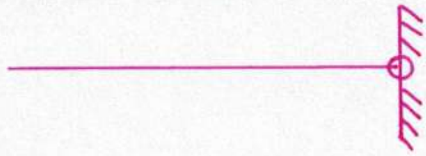
Here, there is no transmitted wave ($A_t = 0$) and reflected wave is π shifted from the incident wave and having the same amplitude.

5.) Reflection at a free end:

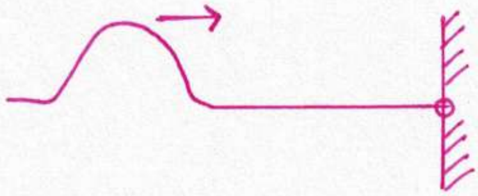
v_2 = extremely large $(= \infty)$

$$A_r = A$$

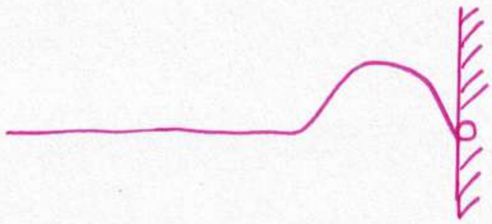
REFLECTION AT A FREE END

1.) 

No pulse

2.) 

Pulse is given

3.) 

Pulse reaches to the ring

4.) 

ring gets uplifted because the particles will take the configuration of the particle which are previous to it.

5.) 

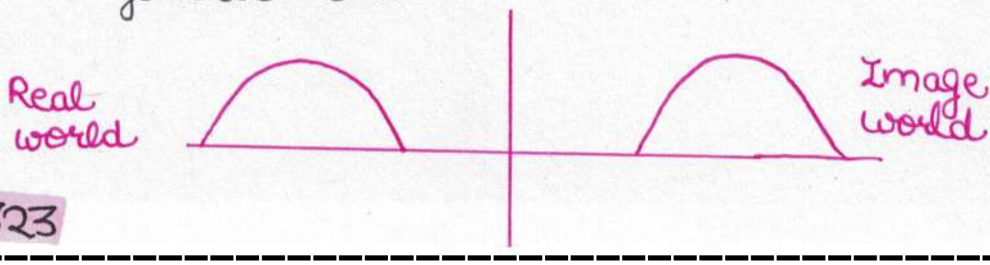
ring tries to take back its shape. Thus ring comes down.

Here, ring has not come down due to gravity as it is a massless ring.

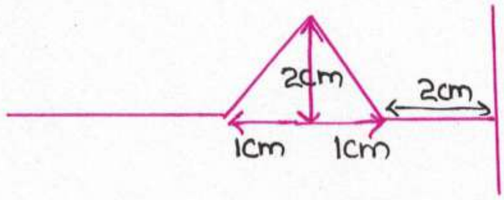
MODELLING OF REFLECTION FROM A FREE END

Step 1: Take the mirror image of the incident pulse assuming free end as the mirror.

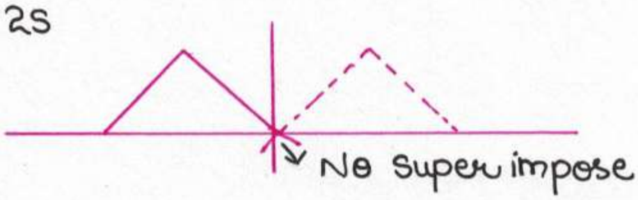
Step 2: Superpose the incident pulse with the pulse coming from image world to get the shape of string at any general time.



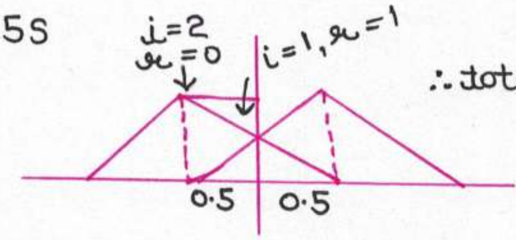
Que.) Sketch the shape of the rop at
(i) $t = 2s$, (ii) $t = 2.5s$, (iii) $t = 3s$, (iv) $t = 3.5s$
(v) $t = 4s$



(i) $t = 2s$

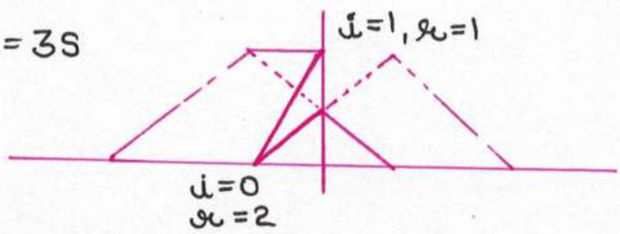


(ii) $t = 2.5s$

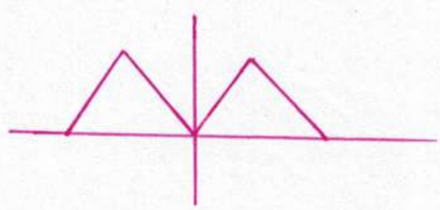


$\therefore$ total = 2

(iii) $t = 3s$

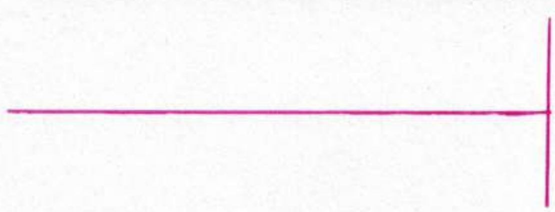


(iv) $t = 4s$



REFLECTION FROM A FIXED END

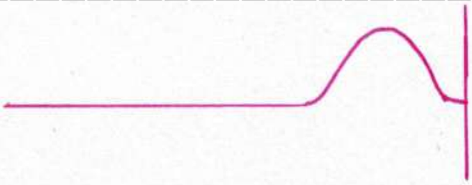
1:)



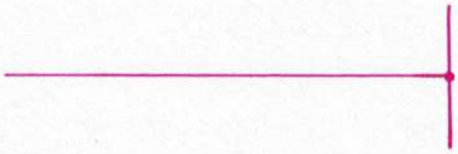
No pulse is there

2:)

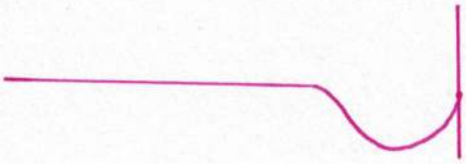
Pulse is given

3.) 

Pulse reaches to the end

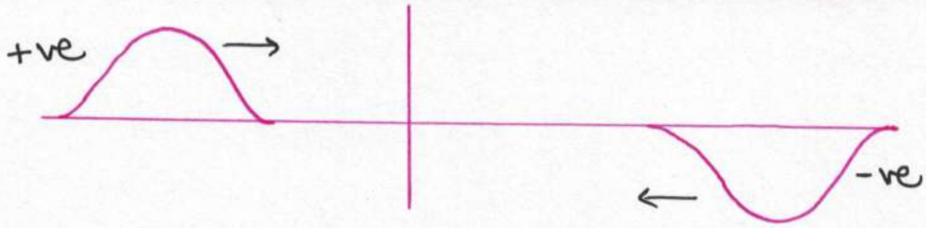
4.) 

As the end is fixed it pulls the rope & therefore it will come down but it will have max. KE & PE = 0

5.) 

As K.E. was max., the end will push it & KE will change into P.E.

MODELLING OF REFLECTION FROM A FIXED END



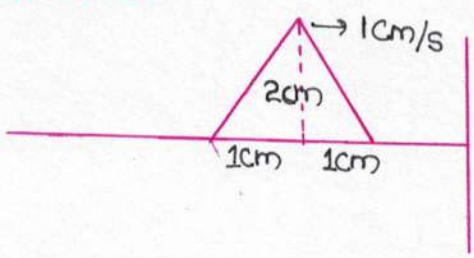
Step 1.) Take the image of the incident pulse about the fixed point in the image world.

Step 2.) Suppose the incident pulse with the pulse coming from image world to get the shape of string. time.

Que.) Sketch the shape of the pulse at

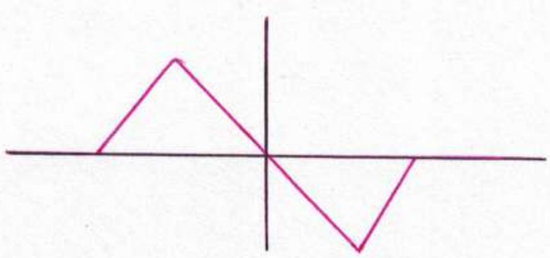
(i) $t = 2s$ (ii) $t = 2.5s$ (iii) $t = 3s$ (iv) $t = 4s$

(v) $t = 3.5s$

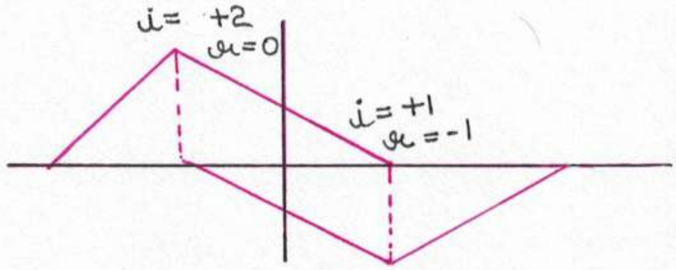


incident reflected sketch

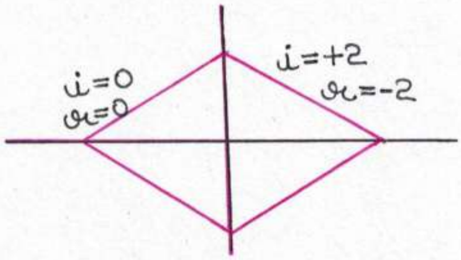
(i) $t = 2s$



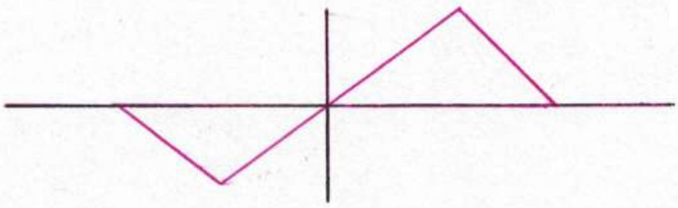
(ii) $t = 2.5$ s



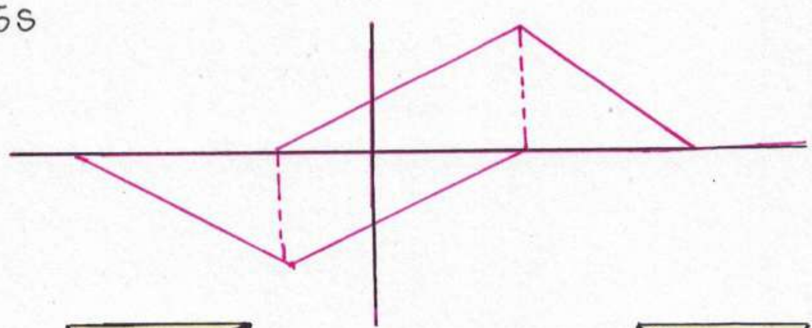
(iii) $t = 3$ s



(iv) $t = 4$ s



(v) $t = 3.5$ s



STATIONARY WAVES

When two waves of equal amplitude and frequency travelling in opposite directions superpose, stationary waves are formed.

For eg: Consider the two waves on a string given by

$$y_1 = A\sin(Kx - \omega t)$$
$$y_2 = A\sin(Kx + \omega t)$$

Superpose, then the resultant displacement is given by

$$y = y_1 + y_2$$
$$= 2A\sin(Kx)\cos(\omega t)$$

We can observe that for the points where $\sin(Kx) = 0$, the particle will not move at all irrespective of time. Such stationary points are called nodes.

$$\sin(Kx) = 0$$
$$Kx = n\pi$$
$$\frac{2\cancel{\pi}}{\lambda}x = n\cancel{\pi}$$
$$x = \frac{n\lambda}{2} \qquad \left(0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}\right)$$

NOTE : 1.) For stationary waves there can be a phase difference b/w two waves.

$$y_1 = A\sin(Kx+\omega t) \text{ and } y_2 = A\sin\left(Kx-\omega t+\frac{\pi}{4}\right)$$
$$y = 2A\sin\left(Kx+\frac{\pi}{8}\right)\cos\left(\omega t-\frac{\pi}{8}\right)$$

2.) We see that node to node distance is $\lambda/2$.

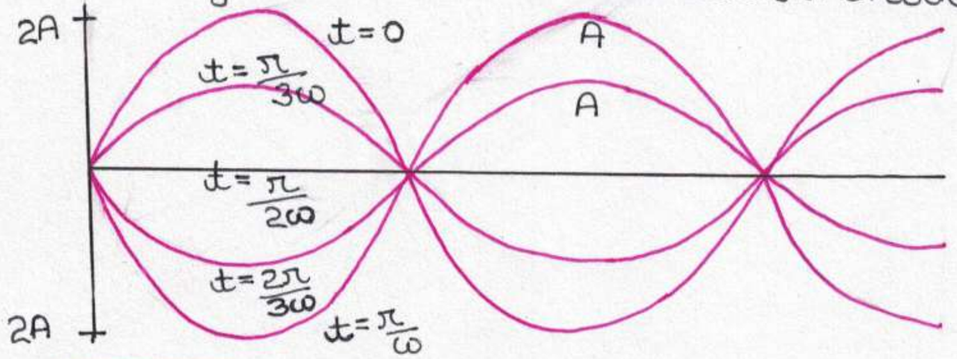
3.) We also see that certain points where $\sin(Kx) = 1$, will oscillate with largest possible amplitude ($2A$). Such points are called anti-nodes.

For anti-nodes,

$$\sin(Kx) = \pm 1$$
$$Kx = (2n+1)\frac{\pi}{2}$$
$$\frac{2\cancel{\pi}}{\lambda}x = (2n+1)\frac{\cancel{\pi}}{2}$$
$$x = (2n+1)\frac{\lambda}{4}$$
$$\left(x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}\right)$$

4.) Energy is not transported along the string to the right or to the left because energy can not flow past the nodal points in the string which are permanently at rest.

Distance b/w any two successive antinodes is $\lambda/2$ and that b/w any node and the nearest antinode is $\lambda/4$.



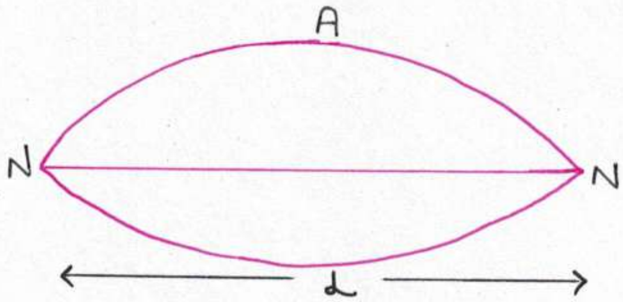
NATURAL FREQUENCIES & RESONANCE

There exists certain frequencies at which if the energy supplied to the system, the system begins to oscillate with large amplitudes. These frequencies are called the natural frequencies or the resonance frequencies of the system.

NATURAL FREQUENCIES OF A STRING FIXED AT BOTH ENDS

Resonance: Driving an oscillator as a characteristic or natural frequency that gives maximum energy transfer from the driving source to the oscillator.

SINGLE LOOP (GUITAR STRING)



$L = \frac{\lambda}{2}$ and $v = \lambda f_1$

$\lambda = \frac{v}{f_1}$

$L = \frac{v}{2f_1}$

$f_1 = \frac{v}{2L}$

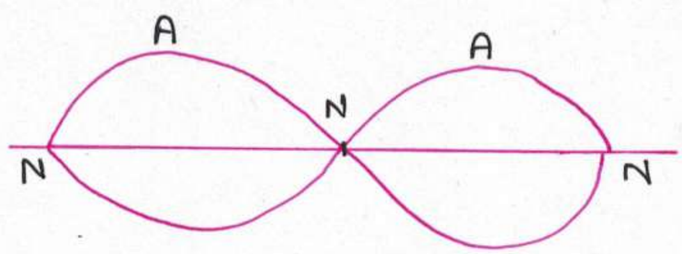
From this, we can easily understand that, guitar will sound shriller, if

(i) tension is increased

(ii) oscillating length is decreased.

(iii) using a thinner string (μ will decrease)

DOUBLE LOOP OSCILLATION



$$L = \frac{2\lambda}{2}$$

$$= \frac{2v}{2f_2}$$

$$\boxed{f_2 = \frac{2v}{2L}}$$

For n loops

$$L = \frac{n\lambda}{2}$$

$$\boxed{f_n = \frac{nv}{2L}}$$

FUNDAMENTAL FREQUENCY

The least frequency for which resonance is observed is called the fundamental frequency of the system.

For string fixed at both ends

$$\text{fundamental frequency} = \frac{v}{2L}$$

HARMONIC NUMBER

The rate of the given frequency to the fundamental frequency is called harmonic no. of that frequency.

fundamental frequency = 50Hz

250 Hz $\longrightarrow$ 5^{th} Harmonic No.

OVERTONE NUMBER

If all the natural frequencies of a system are arranged in ascending order and numbered beginning with zero, then the natural no. corresponding to each frequency is called its overtone number.

For eg - If a string fixed at both ends has natural frequency 50 Hz, then 250 Hz will be called 4^{th} overtone no.

50, 100, 250, 200, 250 ------

0, 1, 2, 3, 4 -------

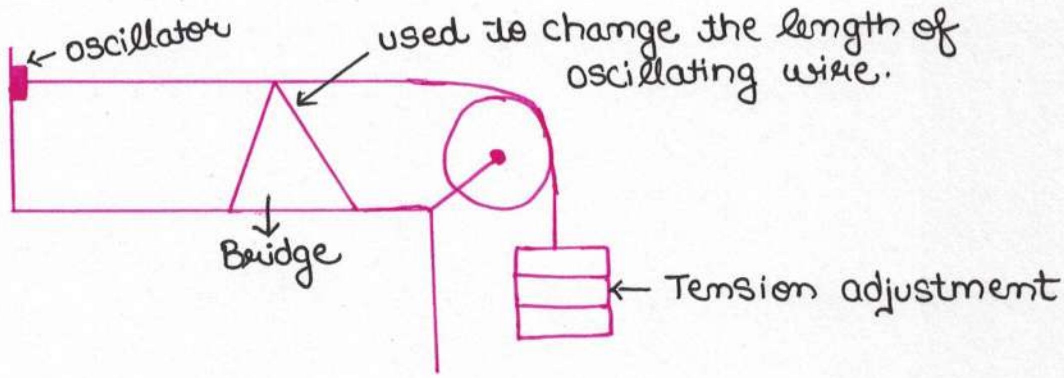
NOTE: For string fixed at both end, we can see that overtone no. is 1 less than the harmonic number.

Que) Two wires of identical diameter, tension, μ are arranged on two sonometers. Find the ratio of their lengths $l_1 : l_2$ if 5^{th} overtone frequency of first string is same as 3^{rd} harmonic frequency of the second string.

$$\frac{6\cancel{v}}{\cancel{2}l_1} = \frac{3\cancel{v}}{2l_2}$$

$$\frac{l_1}{l_2} = \frac{2}{1}$$

Sonometer

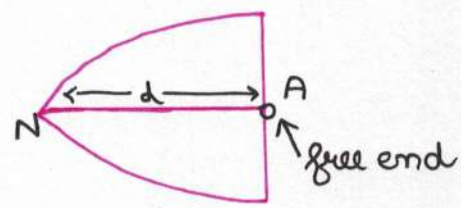


STATIONARY WAVES IN A WIRE FIXED AT ONE END, FREE AT THE OTHER

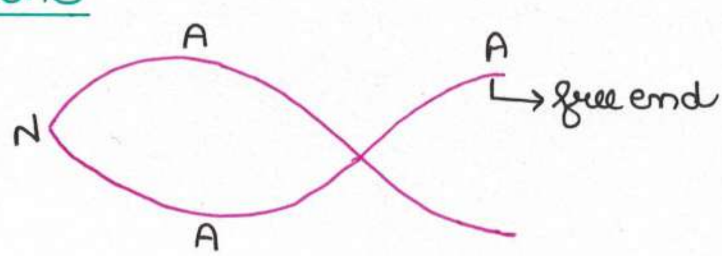
FUNDAMENTAL MODE

$$l = \frac{\lambda}{4} = \frac{v}{4f_0}$$

$$f_0 = \frac{v}{4l}$$



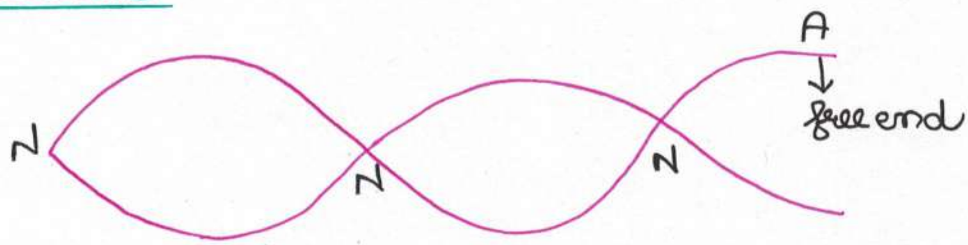
1st Overtone



$$l = \frac{3\lambda}{4} = \frac{3v}{4f_1}$$

$$f_1 = \frac{3v}{4L}$$

2nd Overtone



$$L = \frac{5\lambda}{4} = \frac{5v}{4f_2}$$

$$f_2 = \frac{5v}{4L}$$

$$f_n = (2n+1)\frac{v}{4L}$$

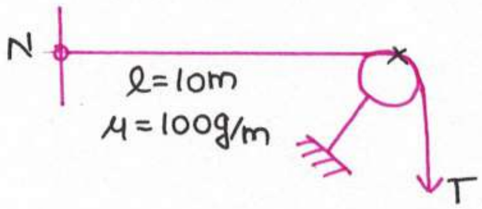
NOTE: n is the overtone no. and $(2n+1)$ is the harmonic number.

Que.) 3rd overtone of a string fixed at one end is same as 6th overtone of a string fixed at both ends. What is the ratio of their lengths if the tension in the first string is 4 times that of the second and μ is same.

$$\frac{\cancel{7} \times 2\cancel{v}}{4L_1} = \frac{\cancel{7}\cancel{v}}{2L_2}$$

$$\frac{L_1}{L_2} = \frac{1}{1}$$

Que.) Find the permissible values of tension for which, we may be able to demonstrate a neat resonance.



$\nu = 20\text{Hz}$

$$T = \frac{4l^2 f^2 \mu}{n^2}$$

$$T = \frac{4 \times 100 \times 40\cancel{0}}{1\cancel{0}\, n^2} = \frac{16000}{n^2}$$

$T = 16000N,\ 4000,\ ------$

Que.) A free violin string sounds a note A (440Hz), where should one put one's finger so as to sound a note C (528 Hz), if the length of the string is 50cm.

$$440 = \frac{v}{4l_A} \quad -(1)$$

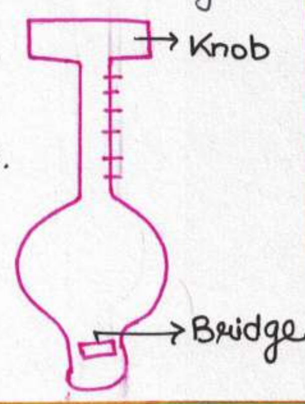
$$528 = \frac{v}{4l_C} \quad -(2)$$

$$\frac{440}{528} = \frac{l_C}{l_A}$$

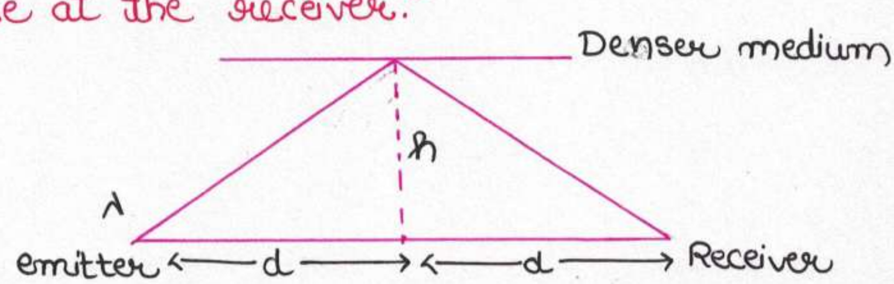
$$l_C = \frac{440}{528} \times 50 = 41.6cm$$

NOTE: In all musical instruments, only one loop is formed and they are cases of a string fixed at both ends.

As the length of oscillating string changes, fundamental frequency changes.



Que.) Find the max. wavelength for which there is a destructive interference at the receiver.



For destructive,

$$\Delta x = n\lambda$$

(path difference)

$$2\sqrt{h^2+d^2} - 2d = n\lambda$$

$$\lambda_{max} = 2\left(\sqrt{h^2+d^2} - d\right)$$

NOTE: Because of the π shift in one of the interfering waves, the condition for maxima and minima are interchanged.

VELOCITY OF A LONGITUDINAL WAVE

Consider a layer of natural thickness dx in a pipe as shown in the fig. Let excess pressure of the layer at the natural coordinate x be P_e and that at at $(x+dx)$ be $P_e + dP_e$. Further let y be the displacement at x and $(y+dy)$ be that at $(x+dx)$.

Now applying Newton's second law to this thickness of air,

$$P_e S - (P_e + dP_e) S = \rho S dx \cdot \frac{\partial^2 y}{\partial t^2} \quad \text{(S is cross-sectional area)}$$

$$\frac{-dP_e}{dx} = \rho \frac{\partial^2 y}{\partial t^2} \quad \text{— (1)}$$

Now the bulk modulus of the air is given by

$$B = \frac{-dP}{(dv/v)}$$

In this case,

$$dP = P_e$$
$$v = S dx$$
$$dv = S dy$$
$$\frac{dv}{v} = \frac{\partial y}{\partial x}$$

$$B = \frac{-P_e}{(\partial y/\partial x)}$$

$$P_e = -B\left(\frac{\partial y}{\partial x}\right) \quad \text{— (2)}$$

$$\frac{\partial P_e}{\partial x} = -B\frac{\partial^2 y}{\partial x^2} \quad \text{— (3)}$$

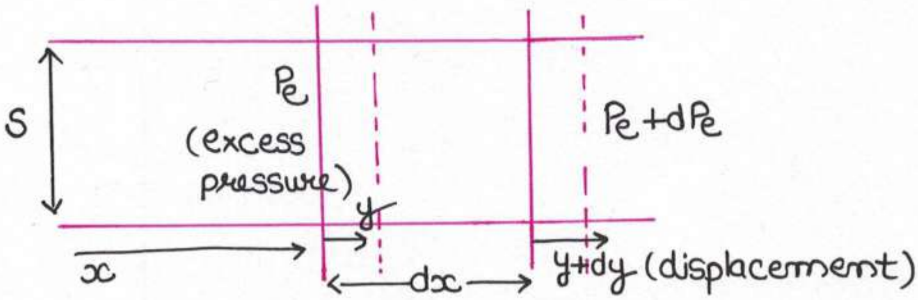
Using (3) in (1)

$$B\frac{\partial^2 y}{\partial x^2} = \rho\frac{\partial^2 y}{\partial t^2}$$

Comparing the above eqn with,

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

$$\therefore v = \sqrt{\frac{B}{\rho}}$$



Que.) Prove that for adiabatic process, $B = \gamma P$

$$PV^{\gamma} = C$$

$$\ln P + \gamma \ln V = \ln C$$

$$\frac{1}{P}dP + \frac{\gamma}{V}dV = 0$$

$$\frac{-(dP)}{(dV/V)} = \gamma P$$

$$B = \gamma P$$

$$\boxed{v = \sqrt{\frac{\gamma P}{\rho}}}$$

$$\boxed{v = \sqrt{\frac{\gamma RT}{M}}}$$

(Rarefactions & compression take place very fast. So we can assume that there is no heat loss.)

In solids, $\boxed{v_{sound} = \sqrt{\frac{Y}{\rho}}}$

RELATION B/W PRESSURE WAVE & DISPLACEMENT WAVE

$$y = A \sin(Kx - \omega t)$$

$$P_e = -B\frac{\partial y}{\partial x}$$

$$= -AKB\cos(Kx - \omega t)$$

$$P_e = BAK \sin\left(Kx - \omega t - \frac{\pi}{2}\right)$$

We see that pressure wave lags behind the displacement wave by a phase of $\pi/2$.

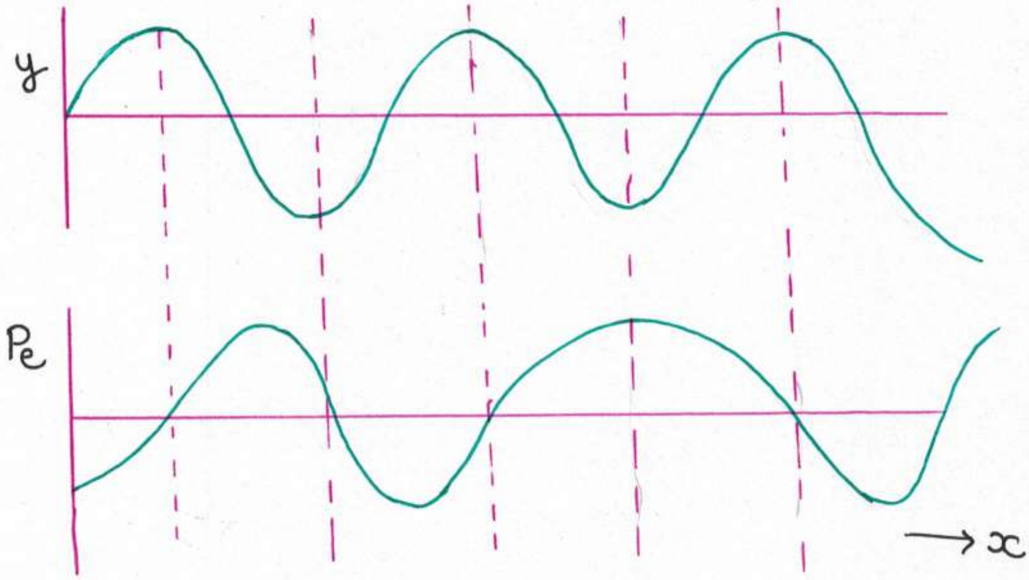
DENSITY WAVE

$$B = \frac{dP}{\left(d\rho/\rho\right)}$$

$$d\rho = \rho \frac{dP}{B}$$

$$\rho_e = \rho \cdot \frac{P_e}{B}$$

we can see that at a point where pressure is max. or min., the displacement is 0, where the displacement is max. or min., the pressure is 0.



RELATION B/w PRESSURE, AMPLITUDE & DISPLACEMENT AMPLITUDE

$$P_{e\,max} = BAK$$

$$I = \frac{1}{2}\rho v G$$

$$I_{avg} = \frac{1}{2}\rho v A^2 \omega^2$$

Que.) Using the above formula, show that average intensity can also be written as $\left(I_{avg} = \frac{P_{e\,max}^2}{2\rho v}\right)$

$$P_{e\,max}^2 = B^2 A^2 K^2$$

$$A^2 = \frac{P_{e\,max}^2}{B^2 K^2}$$

$$I_{avg} = \frac{1}{2} \times \rho \times v \times \frac{P_{e\,max}^2}{B^2 K^2} \omega^2$$

$$= \frac{\rho}{2} \times \frac{v \times P_{e\,max}^2}{B^2} v^2$$

$$I_{avg} = \frac{\cancel{\rho} v P_{e\,max}^2}{2 B^{\cancel{2}}} \times \frac{\cancel{B}}{\cancel{\rho}} \qquad \left(v^2 = \frac{B}{\rho}\right)$$

$$I_{avg} = \frac{v\, P_{e\,max}^2}{2 v^2 \rho}$$

LOUDNESS OF A SOUND

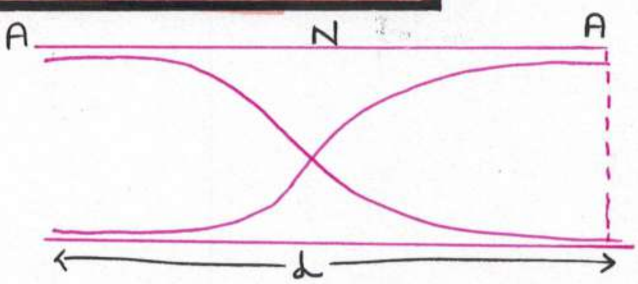
$$\beta = 10 \log \left(\frac{I}{I_0}\right) \quad \text{decibles (dB)}$$

$$I_0 = 10^{-12}\, W/m^2$$

RESONANCE IN ORGAN PIPES

In an organ pipe open end behaves like a free end because particles can easily oscillate in large amplitudes and the close end behave like a fixed end because the particle near the closed end cannot move.

PIPE OPEN AT BOTH END



(open end so $P_e = 0$)

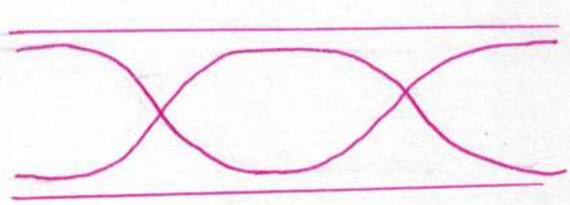
$\therefore$ displacement would be max

$$L = \frac{\lambda}{2} = \frac{v}{2f}$$

$$v = f\lambda$$

$$f_0 = \frac{v}{2L} \quad \text{(fundamental frequency) [for 1 loop]}$$

For 2 loop:



$$L = \frac{2\lambda}{2}$$

$$f_1 = \frac{2v}{2L}$$

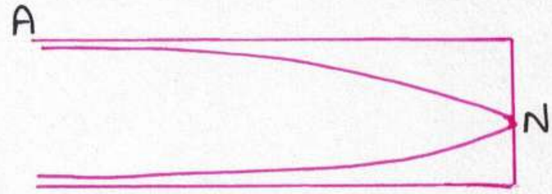
$$f = \frac{nv}{2l}$$

Here n is the harmonic no. and n also represent the no. of nodes.

PITCH PIPE CLOSED AT ONE END

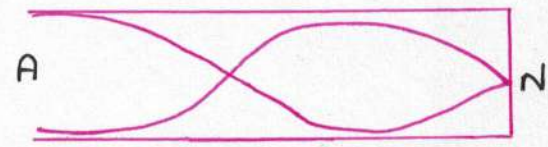
$$l = \frac{\lambda}{4} = \frac{v}{4f}$$

$$f_0 = \frac{v}{4l} \quad \text{(fundamental frequency)}$$



For 2 loops

$$l = \frac{3\lambda}{4} = \frac{3v}{4f}$$

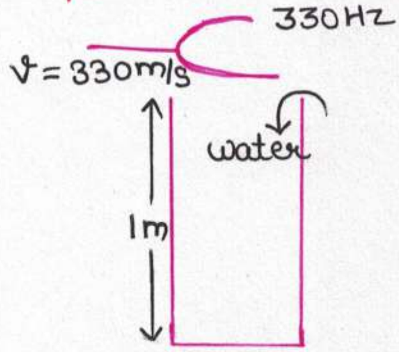


$$\text{or } f_1 = \frac{3v}{4l}$$

$$f = (2n+1)\frac{v}{4l} \qquad (n \text{ is the overtone no.})$$

Que.) A pitch pipe of length 1m is excited using a tuning fork of frequency 330Hz. Now water is gradually poured into the pipe. For what levels of water would you observe resonance

$$l = \frac{(2n+1)\,\cancel{330}}{4 \times \cancel{330}}$$



$$l = \frac{(2n+1)}{4}$$

$$l = \frac{1}{4} \text{ or } \frac{3}{4}\text{m}$$

$\downarrow \qquad \downarrow$

water level $\frac{3}{4}$m or $\frac{1}{4}$m

Que.) An open organ pipe has a fundamental frequency 330Hz. The first overtone of a closed pipe has the same frequency as first overtone of this pipe. How long is each pipe. Take v $v = 330$ m/s.

For open pipe

$$F_0 = 330 = \frac{v}{2l}$$

$$l = \frac{1}{2}\text{m}$$

For closed pipe

$$\frac{3v}{4l} = \frac{v}{L}$$

$$l = \frac{3L}{4} = \frac{3}{4}\times\frac{1}{2} = \frac{3}{8}\text{m}$$

Que.) The first overtone frequency of a stretched string of length 1m is 330 Hz. Its 3rd overtone has same frequency as the 1st overtone of an open pipe. Find the length of open pipe if the velocity of wave on the string is 165 m/s. velocity of sound is 330 m/s.

$$330 = \frac{v}{L}$$

$$L = \frac{\cancel{165}^{1}}{\cancel{330}_{2}} = \frac{1}{2}\text{m} \quad \text{(For string)}$$

For open pipe

$$\frac{2v}{L} = \frac{v}{l}$$

$$l = \frac{v}{330\times 2} = \frac{1}{2}\text{m}$$

END CORRECTION

Practically in pitch pipes, the actual antinode is not formed at the open end but at a further distance 0.3 D away.
This distance is added to the length of the pitch pipe to get the effective length and is known as end correction.

$$e = 0.3D \qquad \text{(D is diameter of the pipe)}$$

Que.) Fundamental frequency of a pitch pipe filled with oxygen is 100 Hz. What will be ~~the~~ its fundamental frequency if it was filled with He at 4 times the temperature?

$$f_{HE} = \frac{v_{HE}}{2L}$$

$$f_{O_2} = \frac{v_{O_2}}{2L}$$

$$\frac{f_O}{f_{He}} = \frac{v_O}{v_{He}}$$

$$\frac{f_O}{f_{He}} = \sqrt{\frac{\gamma_O R T_O M_{He}}{\gamma_{He} R T_{He} M_{O_2}}}$$

$$= \sqrt{\frac{7\times 3}{5\times 5} \times \frac{T_O}{4T_O} \times \frac{4}{32}}$$

$$f_{He} = \frac{20\sqrt{2}}{\sqrt{21}} \times 100$$

BEATS

Consider two oscillations with angular frequencies ω_1 & ω_2 superposing on each other with ω_1 being slightly different from ω_2.

$$y_1 = A \sin(\omega_1 t)$$

$$y_2 = A \sin(\omega_2 t)$$

we can write the resultant oscillation as

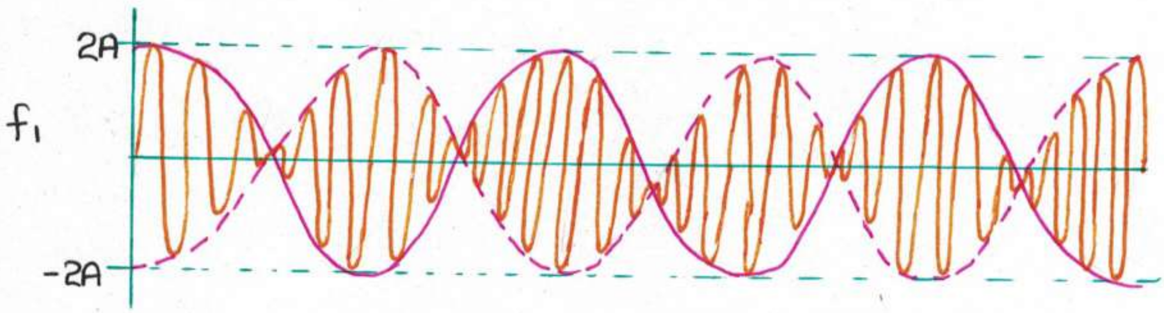
$$y = y_1 + y_2 = 2A \sin\left(\frac{\omega_1+\omega_2}{2}\right)t \cdot \cos\left(\frac{\omega_1-\omega_2}{2}\right)t$$

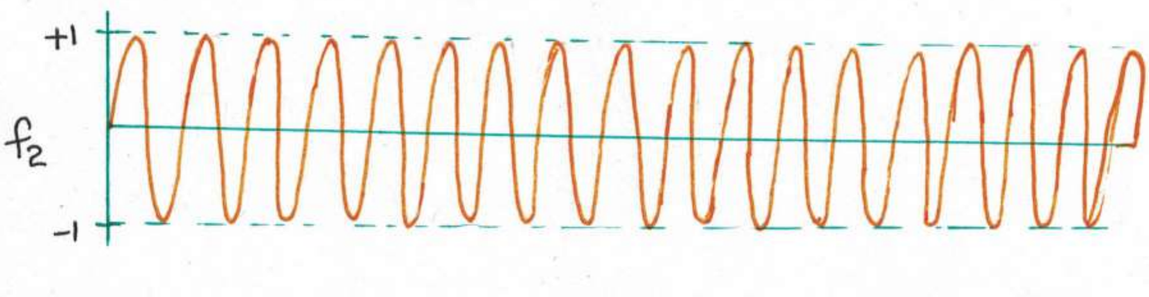
which can be thought of two functions as:

$$f_1 = 2A \cos\left(\left(\frac{\omega_1-\omega_2}{2}\right)t\right)$$

$$f_2 = \sin\left(\left(\frac{\omega_1+\omega_2}{2}\right)t\right)$$

Here the resultant amplitude is 2A because phase difference is 0.





These graphs are drawn relative to each other.

When the two frequencies are superposed, we see that we get a sound wave of variable amplitude which reaches its loudest value whenever the cosine function reaches +1 or -1 (ie twice in every cycle of cos function). This means the no. of times we hear the loudest sound in a second will be double the frequency of cosine function. This frequency is called the beat frequency.

$$\nu_{beat} = \frac{1}{2\pi}\left|\left(\frac{\omega_1-\omega_2}{2}\right)\right| \times 2$$

$$\nu_{beat} = \left|\frac{\omega_1}{2\pi} - \frac{\omega_2}{2\pi}\right|$$

$$\nu_{beat} = \nu_1 - \nu_2$$

We can consider $\cos\theta = -1$ as maxima because there is almost full probability that sin func.ⁿ would have -1 value at that time. So it would make +1 at that time and thus creating the maxima.

NOTE: whenever a tuning fork is loaded with wax its frequency decreases and whenever a tuning fork is filled, it's frequency increases.

Que.) A tuning fork of unknown frequency makes 3 beats per second with standard fork of 348 Hz. When the unknown fork is very slightly loaded with wave, the beat frequency decreases. What is the frequency of unknown fork.

351 (348+3 or 348-3)

we will choose 351 because the difference should decrease when we have loaded the fork with wax.

Que.) A pitch pipe of length L has a beat frequency f with a tuning fork of frequency f_0. If the pitch pipe is heated it is found that once again the beat frequency is f. The initial temp. is T_0 and the length of the pitch pipe is L and molar mass of the gas is M and the gas is diatomic. Develop an equation to solve for final temp. Neglect thermal expansion of the pipe. Pitch pipe is open at both ends. Fundamental mode is being noticed.

$$f + f_0 = \frac{v_2}{2L}$$

$$f - f_0 = \frac{v_1}{2L}$$

$$2L(f - f_0) = \sqrt{\frac{\gamma R T_0}{M}} \quad -①$$

$$2L(f + f_0) = \sqrt{\frac{\gamma R T_F}{M}}$$

divide ① & ②

$$\frac{f + f_0}{f - f_0} = \sqrt{\frac{T_F}{T_0}}$$

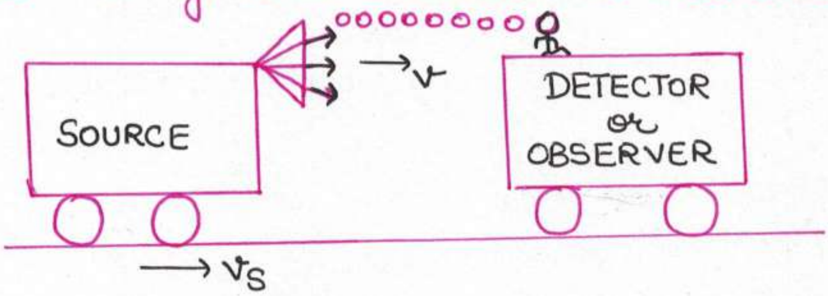
$$T_F = T_0 \left(\frac{f_1 + f_0}{f - f_0}\right)^2$$

NOTE: Human ears cannot detect beat frequency more than 7Hz. This is the reason, why we have taken slightly different 'ω's and not drastically different 'ω's

DOPPLER EFFECT

When a detector detects the second emitted by a moving emitter, or if the detector is itself moving than in general the frequency of the detected sound is different from the frequency of the emitted sound. This is called Doppler effect.

Case 1:
Source is moving and observer is stationary.



Consider a source moving with velocity v_S emitting a sound of frequency f which moves with velocity v relative to the medium.

We can find the wavelength of sound using simple kinematics.

Consider a compression released at this moment, after one time period, it would have travelled a distance (vT) while source would have moved a distance $(v_S T)$ and will be at the verge of releasing another compression.

Thus the wavelength of sound in air will be

$$\lambda' = (v - v_S)T$$

In one second, the compression will move a distance v relative to the observer and thus the ~~#~~ no. of compressions passing the observer is given by

$$f' = \frac{v}{\lambda'} = \frac{v}{(v - v_S)T}$$

or, $\boxed{f' = f\left(\frac{v}{v - v_S}\right)}$

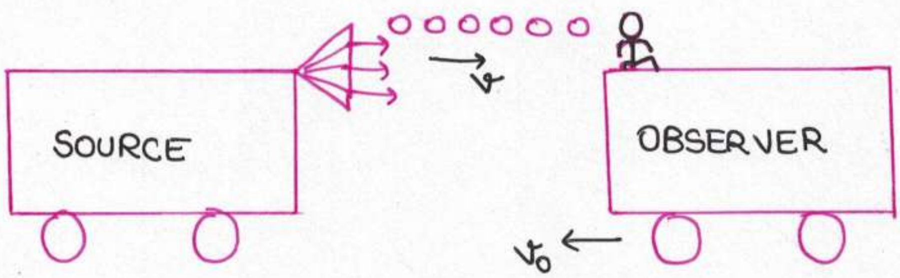
observed frequency

emitted frequency

Similarly, if the source were moving away from observer, it can be readily shown that

$$\left(f' = \frac{f \cdot v}{v + v_S}\right)$$

Case 2:



$$v = f\lambda$$

$$f' = \left(\frac{v + v_0}{\lambda}\right)$$

$$f' = f\left(\frac{v + v_0}{v}\right)$$

Similarly if the observer were moving away from source

$$f' = f\left(\frac{v - v_0}{v}\right)$$

General Case:

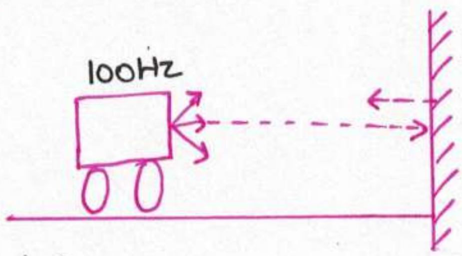
Both observer and source are moving

$$f' = \left(\frac{v \pm v_0}{v \pm v_S}\right)f$$

'+' & '-' are to be decided using common sense.

Consider the motion of one at a time. If due to its motion separation is increasing frequency must decrease and if separation is decreasing, frequency must increase.

Que.) A car travelling at 33 m/s head on towards a hill sounds a horn of frequency 1000 Hz. Find the beat frequency heard by the observer in the car due to reflection from the hill. Given $V_{sound} = 330\,m/s$



$$f' = \left(\frac{v}{v - v_c}\right) f$$

(f' is the frequency by which sound is hitting the hill that is emitted by car)

$$f'' = \left(\frac{v + v_c}{v}\right) f'$$

(f'' is the frequency by which car is receiving the sound emitted by hill.)

$$f_{beat} = f'' - f$$

$$= f\left[\frac{v + v_c}{v - v_c} - 1\right]$$

$$= \left(\frac{2v_c}{v - v_c}\right) f$$

$$= \frac{2 \times 33}{330 - 33} \times 1000$$

$$f_{beat} = \frac{2000}{9}\,Hz$$

Que.) Microwaves emitted from a source beat against the reflected waves from an aeroplane. Find the speed of the aeroplane given that

$\nu_{beat} = 990\,Hz$

$\lambda = 0.1\,m$

$C = 3 \times 10^8\,m/s$

$f_{beat} = \nu_m$

$$\nu = \underset{\substack{\downarrow \\ \text{microwave}}}{\nu_m}\left(\frac{v + v_0}{v}\right) = \nu_m\left(\frac{c + v}{c}\right)$$

$$\nu_r = \nu_i \left(\frac{v}{v - v_s}\right) \Rightarrow \nu_r = \nu_i \left(\frac{c}{c-v}\right) = \left(\frac{c+v}{c-v}\right)\lambda_m$$

$\nu_r \downarrow$ reflected $\nu_i \downarrow$ incident

$$\nu_r - \nu_m = \nu_{beat}$$

$$\left(\frac{c + v - c + v}{c - v}\right)\nu_m = \nu_{beat}$$

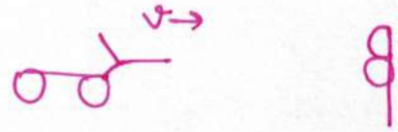
$$\frac{2v\nu_m}{c-v} \approx \frac{2v\nu_m}{c} \approx \nu_{beat}$$

$$\frac{2v}{\cancel{c}} \times \frac{\cancel{c}}{\lambda} = \frac{2v}{\lambda} = \nu_{beat}$$

$$v = \frac{\nu_{beat} \times \lambda}{2}$$

$$v = \frac{990 \times 0.1}{2} = 49.5\ m/s$$

Que.) A person claims that, he perceived the red signal to be green due to doppler effect. Is it possible.
Given that $\lambda_{red} = 600\ nm$
$\lambda_{green} = 500\ nm$



$$f = f_i\left(\frac{v_L + v_o}{v_L}\right)$$ ($v_L \rightarrow$ velocity of light)

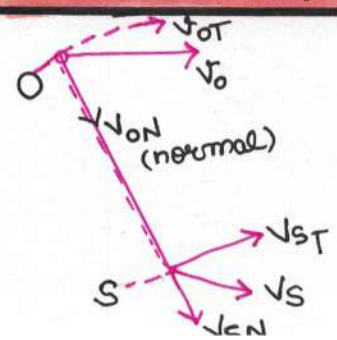
$$\frac{c}{\lambda_{green}} = \frac{c}{\lambda_{red}}\left(\frac{c+v}{c}\right)$$

$$\frac{60\cancel{0}}{50\cancel{0}} = 1 + \frac{v}{c}$$

$$\frac{1}{5} = \frac{v}{c}$$

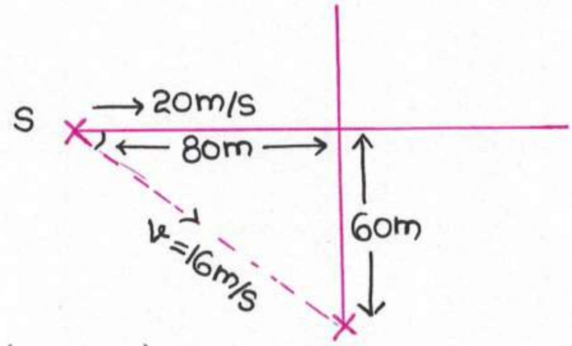
$$v = \frac{c}{5}$$

DOPPLER EFFECT WITH OBLIQUE APPROACH



$$\boxed{\nu' = \left(\frac{v \pm v_{oN}}{v \pm v_{sN}}\right)\nu}$$

Que.) A car approaching a crossing at 20 m/s sounds a horn of 500Hz when 80m away from the crossing. A man is standing 60m away from the crossing on a ⊥ road. What is the frequency heard by the man?



$$f_h = f\left(\frac{v}{v - v_s}\right)$$

$$f_h = 500\left(\frac{300}{330-16}\right)$$

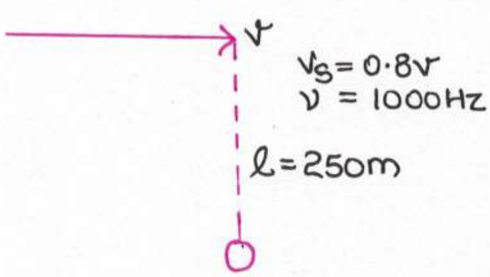
$$f_h = \frac{500 \times 330}{314}$$

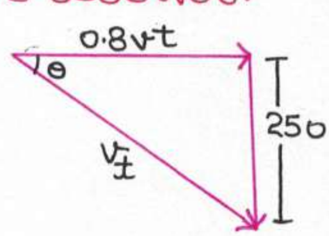
$$f_h = 525.477\ \text{Hz}$$

DOPPLER EFFECT when the velocity of Source or Observer is Comparable to that of sound

This case is similar to the oblique approach case, however the position of the source should be taken at the time of emitting and the position of the observer should be taken at time of hearing.

Que.) A source travelling with 0.8 mach sounds a horn of frequency 100 Hz as shown. An observer is located 250m away from its line of travel. Find the observed frequency when the Source is nearest to the observer.





$$f_h = 1000\left(\frac{v}{v - 0.64v}\right)$$

$$f_h = \frac{1000 \times 100\cancel{v}}{0.36\cancel{v}}$$

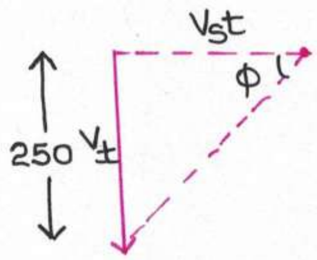
$$f_h = 2777.77\ \text{Hz}$$

Que.) In the previous question what will be the distance b/w the source & the observer when the observed frequency is same as the emitted frequency.

$$f_n = f\left(\frac{v}{v - v_s \cos\theta}\right)$$

$$\therefore \theta = \frac{\pi}{2}$$

$$\tan\phi = \frac{v\cancel{t}}{v_s\cancel{t}} = \frac{v}{0.8v} = \frac{.5}{4}$$



Distance of observer from source $= \dfrac{250}{\sin\phi}$

$$= \frac{\overset{50}{\cancel{250}}}{\cancel{5}}\sqrt{41}$$

$$= 50\sqrt{41}$$

Doppler Effect when the wind is blowing :

When wind is blowing, we can convert all the velocities (source and observer) relative to the speed of wind and apply the doppler effect as usual.

Que.) Find the observed frequency.

$v_w = 99\,m/s \longrightarrow$

$\longrightarrow 33\,m/s$ $33\,m/s \longleftarrow$

Source   observer

$\longleftarrow 66\,m/s$ $\longleftarrow 132\,m/s$

$\nu = 500\,Hz$, $V_{sound} = 330\,m/s$

$$f_0 = f\left(\frac{V_{sw} + V_{ow}}{V_{sw} - V_{w}}\right)$$

$$= 500\left(\frac{330 + 132}{330 + 66}\right)$$

Que.) A bomb blast happens at a distance l from an observer while the wind is blowing perpendicular to the line joining person to the blast side as shown in fig. After how much time, will the person hear the blast?

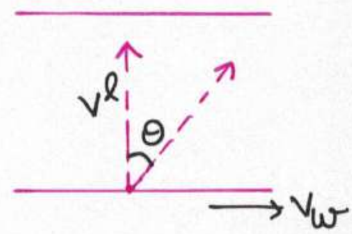
$\sin\theta = \dfrac{V_w}{V_{Br}}$ (by taking it as river boat problem)

$$t = \frac{l}{V_{Br}\cdot\cos\theta} = \frac{l}{\sqrt{v^2 - V_w^2}}$$

Que.) In previous que. at what point on AB will the blast be heard first.

required distance $= \frac{l}{v} \times v_w$

↑ time ↑ velocity



Que.) A person standing on the shore throws a stone with velocity u at an angle of 45° ⊥ to the flow of the river. After how much time will the ripples reach his feet. V_r is the velocity of river & v is velocity of ripples.

$$T_{stone} = \frac{2u\sin\theta}{g}, \quad R = \frac{u^2\sin 2\theta}{g}$$

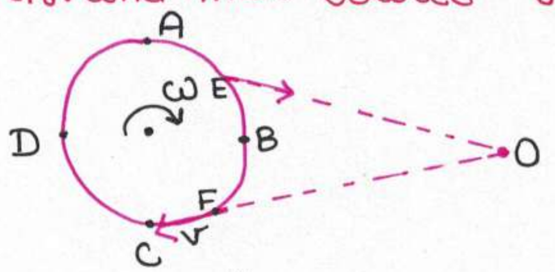
$$T_{ripple} = \frac{R}{\sqrt{v^2 - V_r^2}}$$

$$\text{Total time} = \frac{2u\sin\theta}{g} + \frac{u^2\sin 2\theta}{g\sqrt{v^2 - V_r^2}}$$

Que.) In previous que., the frequency of throwing stone is 0.1Hz. Will the frequency with which ripples touch the feet be same or not?

Yes, it will be the same.

Que.) Find the points where intensity will be max. & frequency will be min. and max. Source is to be put on the point.



B → max. intensity

E → max. frequency

F → min. frequency

$$f' = f\left(\frac{v}{v - v_s\cos\theta}\right)$$

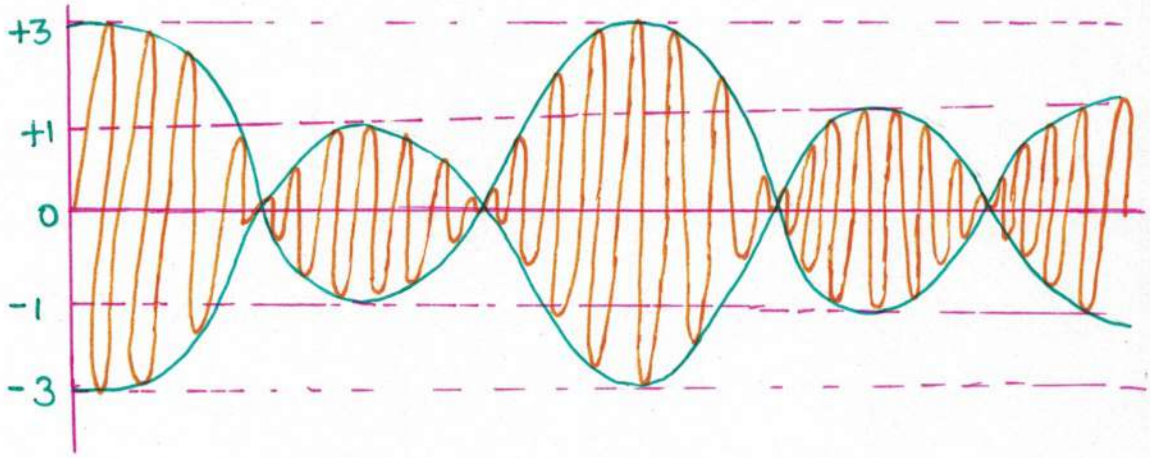
Que.) If the waves originating from three sources of sound having equal amplitude and frequencies 300, 301 and 302 Hz respectively. Superpose, the no. of beats heard per second will be ______.

As frequencies are in AP, ω's will also be in AP.

$$A \sin(\omega t) + A \sin(\omega + \Delta\omega)t + A \sin(\omega - \Delta\omega)t$$

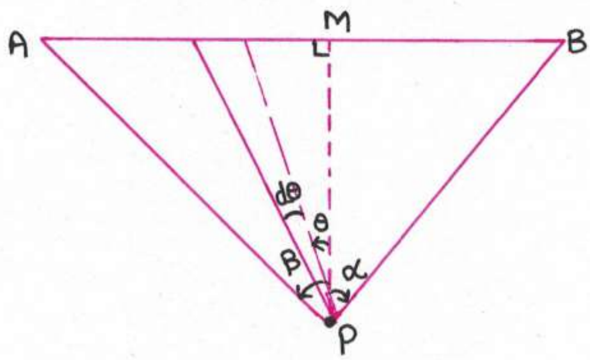
$$= A \sin(\omega t) + 2A \sin(\omega t) \cdot \cos(\omega t)$$

$$= A \sin(\omega t) \cdot [1 + 2 \cos(\Delta\omega t)]$$



NOTE : For a given fundamental frequency f, $2f$ is called upper octave of f and $f/2$ is called the lower octave of f.

FIELD DUE TO A LINE CHARGE



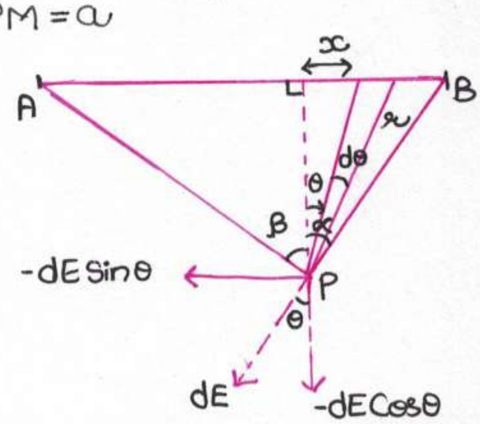
$\lambda, \alpha, \beta, a$ (are given)

AB is a line charge and charge per unit length is λ. We have to find electric field at point P. $PM = a$

$$x = a \tan\theta$$

$$dx = a \sec^2\theta . d\theta$$

$$r = a \sec\theta$$



$$dE_x = - dE \sin\theta$$

$$dE_y = - dE \cos\theta$$

$$dE = \frac{K dq}{r^2}$$

$$dE = \frac{K\lambda \cancel{a} \sec^{\cancel{2}}\theta . d\theta}{a^{\cancel{2}} \cancel{\sec^2}\theta} = \frac{K\lambda}{a} d\theta$$

$$dE_x = \frac{-K\lambda}{a} \sin\theta . d\theta$$

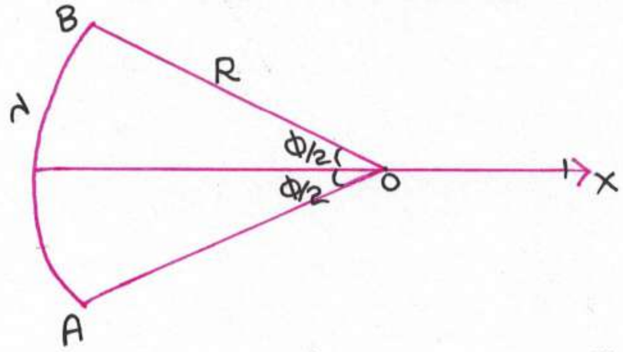
$$\int_{-\beta}^{\alpha} dE_x = \frac{-K\lambda}{a} \int_{-\beta}^{\alpha} \sin\theta . d\theta$$

$$\boxed{dE_x = \frac{K\lambda}{a}(\cos\alpha - \cos\beta)}$$

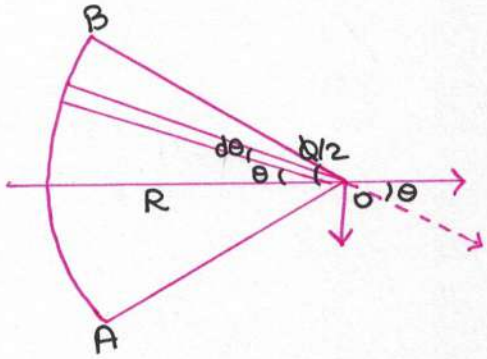
$$\int_{-\beta}^{\alpha} dE_y = \frac{-K\lambda}{a} \int_{-\beta}^{\alpha} \cos\theta . d\theta$$

$$\boxed{dE_y = \frac{-K\lambda}{a}(\sin\alpha + \sin\beta)}$$

FIELD DUE TO ARC



AB is an arc and charge per unit length is λ. Find electric field at point O. OB = R



$$dE = \frac{K dq}{r^2}$$

$$= \frac{K \lambda \cancel{R} d\theta}{R^{\cancel{2}}} = \frac{K\lambda d\theta}{R}$$

$$dE_y = \frac{K\lambda d\theta}{R} \cos\theta$$

$$\int_0^{\phi/2} dE_y = \int_0^{\phi/2} \frac{K\lambda d\theta}{R} . \cos\theta$$

$$E_y = \frac{2K\lambda}{R} \sin\frac{\phi}{2}$$

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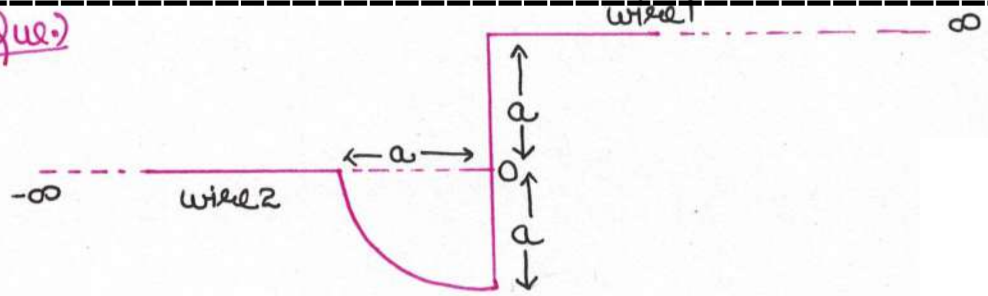
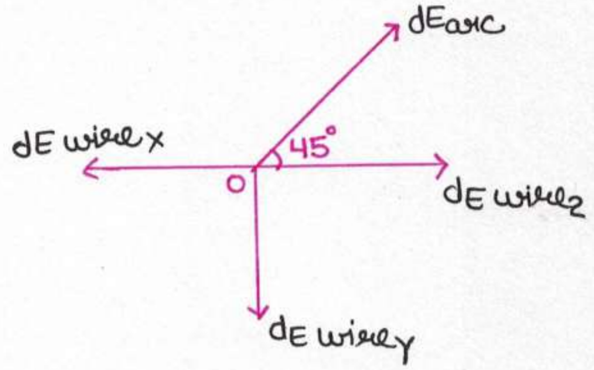
Que.)



$$dE_{arc} = \frac{2K\lambda}{R}\sin 45^\circ$$

$$= \frac{\sqrt{2}K\lambda}{a}$$



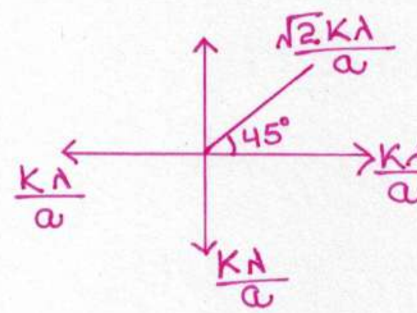
$$dE_{wire\,x} = \frac{K\lambda}{a}(0-1)$$

$$= \frac{-K\lambda}{a}$$

$(\alpha = 90^\circ, \beta = 0^\circ)$

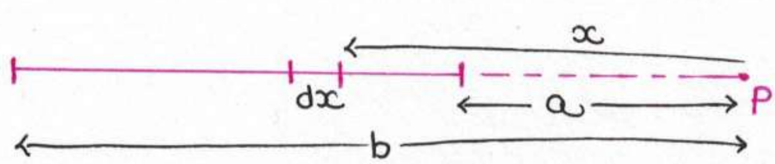
$$dE_{wire\,Y} = -\frac{K\lambda}{a}(1) = -\frac{K\lambda}{a}$$

$$dE_{wire\,2} = \frac{-K\lambda}{a}\left(-\frac{1}{a}\right) = \frac{K\lambda}{a} \quad (b=\infty, a=a)$$



$$\therefore E_{Total} = +\frac{K\lambda}{a}$$

ELECTRIC FIELD DUE TO A LINE (POINT IS ON THE LINE)



λ = point charge

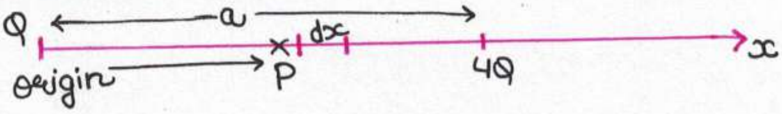
$$dE = \frac{K\lambda dx}{x^2}$$

$$\int_a^b dE = \int_a^b \frac{K\lambda}{x^2}\cdot dx$$

$$= -K\lambda\left(\frac{1}{b}-\frac{1}{a}\right)$$

$$\boxed{E = -K\lambda\left(\frac{1}{b}-\frac{1}{a}\right)}$$

GRAPH ANALYSIS



$$E = \frac{KQ}{x^2} + \frac{4KQ}{(x-a)^2}$$

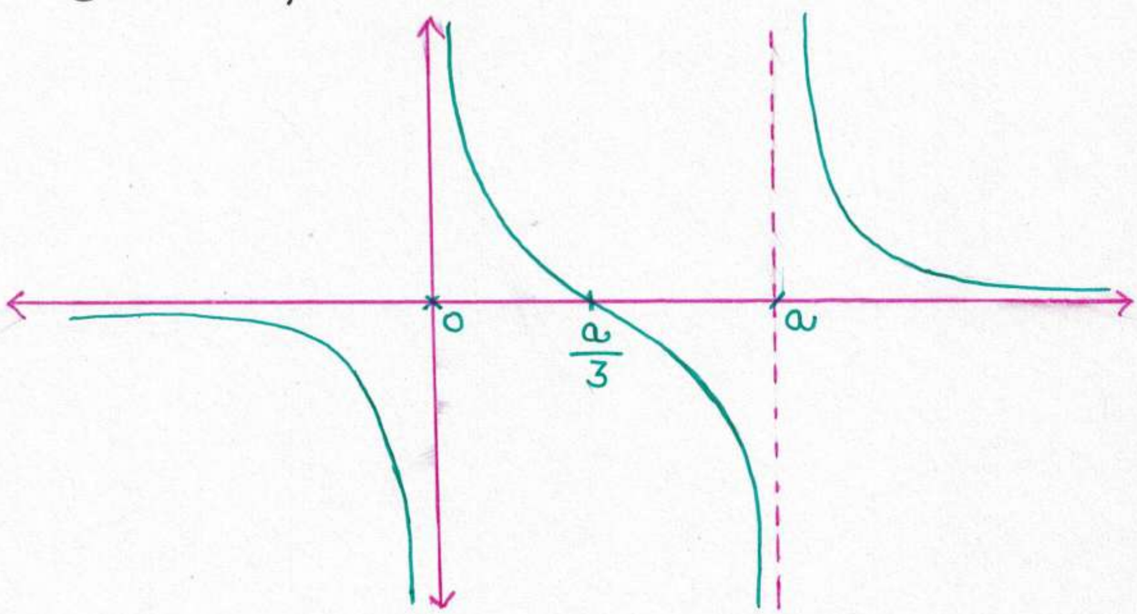
@ $x = -\infty$, $E = 0$

@ $x = 0$, $E = -\infty$

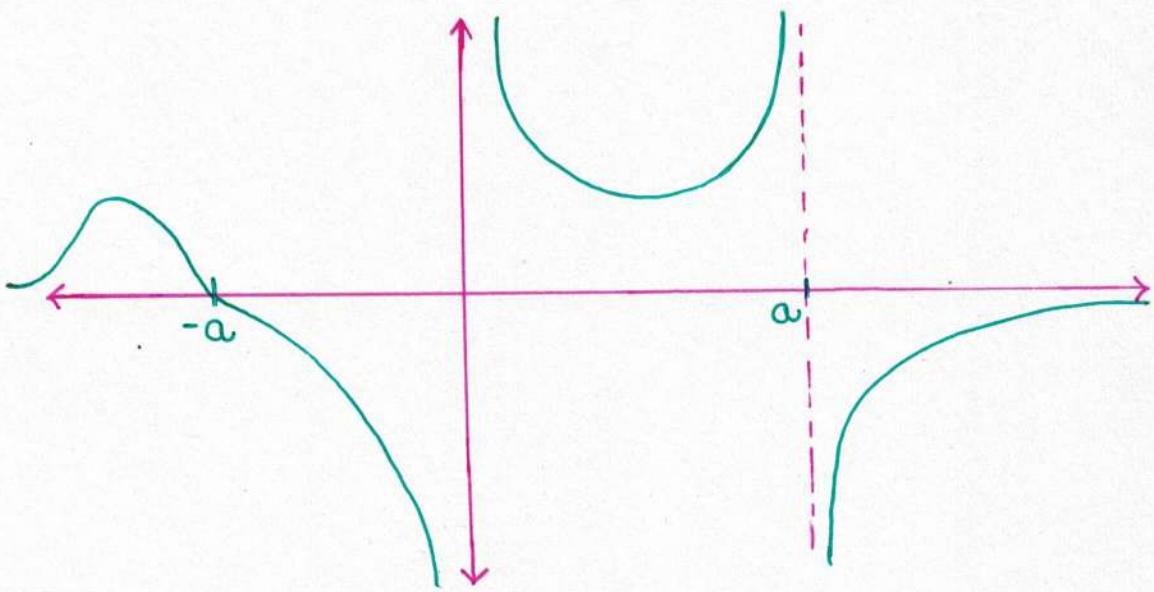
@ $x = \frac{a}{3}$, $E = 0$

@ $x = a$, $E = \infty$

@ $x = \infty$, $E = 0$



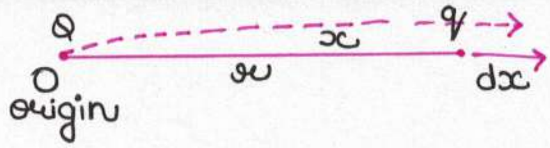
Que.) Plot the E-x graph for charges q and -4Q placed at a distance a.



ELECTRIC POTENTIAL ENERGY

The negative of the work done by the electric field is called electric Potential.

ELECTRIC POTENTIAL ENERGY DUE TO POINT CHAGE



$$dW_e = K\left(\frac{Qq}{x^2}\right)dx = -dV$$

$$(\Delta V = -F.dr)$$

$$\int_{U}^{U_\infty} dU = \int_{r}^{\infty} -KQq\, x^{-2}.dx$$

$$U = U_\infty + \frac{KQq}{r}$$

by convention, $U_\infty = 0$

$$\boxed{\therefore U = \frac{KQq}{r}}$$

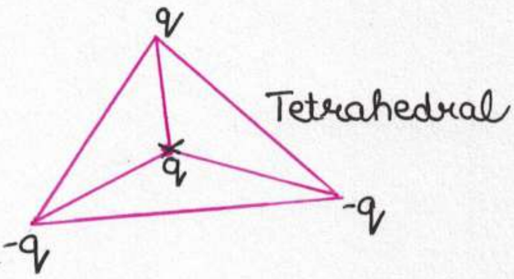
For more than two point particles,

$$U = \sum \frac{KQ_i q_j}{r_{ij}}$$ (take every pair once using sign convention)

Que.) 4 point charges of equal magnitude 'q' two positive & two negative have equal separation 'a' between them. What is the potential energy of system.

Total pairs $= {}^4C_2 = 6$

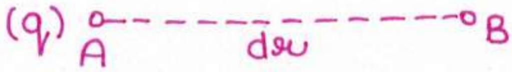
$$U = -\frac{2q^2K}{a}$$



NOTE: When the Configuration of the System is changed and U_i and U_f are initial and final potential energies, then,

$$W_{extagent} = U_f - U_i$$

$$W_{electric\ field} = U_i - U_f$$



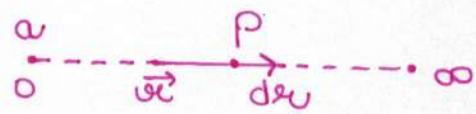
$\Delta U = W_{ext.}$

$\Delta V = \dfrac{\Delta U}{q} = -\dfrac{\vec{F}.d\vec{r}}{q}$

$= -\left(\dfrac{\vec{F}}{q}\right).d\vec{r} = -\vec{E}.d\vec{r}$

$$\boxed{d\vec{V} = -\vec{E}.d\vec{r}}$$

Def:- Electric potential is negative of the work done by the electric field per unit charge.



$\int_{V_\infty}^{V} dV = \int_{\infty}^{r} -\vec{E}.d\vec{r}$

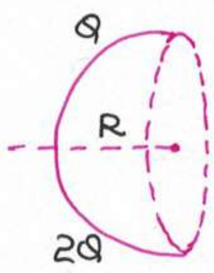
$V = \dfrac{Kq}{r} + V_\infty$

by convention, $V_\infty = 0$

$V = \dfrac{Kq}{r}$

$$\boxed{\begin{aligned} &\Delta V = -(E_x dx + E_y.dy + E_z.dz) \\ &E_x = -\frac{\delta V}{\delta x}, \; E_y = -\frac{\delta V}{\delta y}, \; E_z = -\frac{\delta V}{\delta z} \end{aligned}}$$

Que.) A hollow hemisphere is given with charge Q on half of it & 2Q on the other half. Find potential at the center.



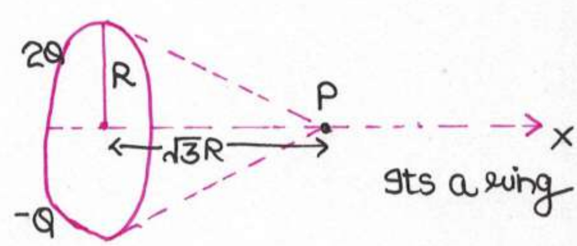
$V = \dfrac{QK}{r} + \dfrac{2QK}{r}$

$V = \dfrac{3KQ}{r}$

Que.)

$V_P = \dfrac{2KQ}{2R} - \dfrac{KQ}{2R}$

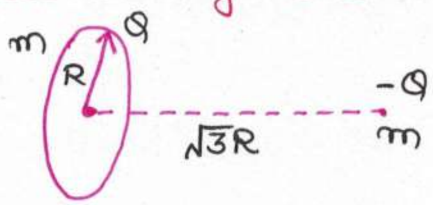
$V_P = \dfrac{KQ}{2R}$



$$E_{px} = \frac{QK\sqrt{3}R}{(2R)^3} = \frac{\sqrt{3}QK\cancel{R}}{8R^{\cancel{3}2}} = \frac{\sqrt{3}QK}{8R^2}$$

or, $$E_{px} = -\frac{\delta V}{\delta x}$$

Que.) The ring and the particle each carrying a charge Q, is released in free space. Find the speed of the particle when it will pass through the centre of the ring.



<u>COM</u> $v_r = v_p$

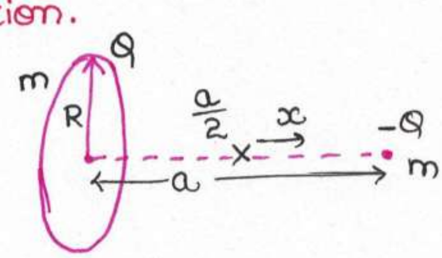
$r \rightarrow$ ring
$P \rightarrow$ particle

<u>COME</u> $$2\left(\frac{1}{2}mv^2\right) - \frac{KQ^2}{R} = 0 + K\left(\frac{-Q^2}{2R}\right)$$

$$mv^2 = \frac{KQ^2}{2R}$$

$$v = Q\sqrt{\frac{K}{2mR}}$$

Que.) The situation is same as in the previous case but this time distance b/w them is $a << R$. Calculate the time period of oscillation.



$$F = \frac{-Q^2K}{(R)^3}(2x)$$

$$T = 2\pi\sqrt{\frac{mR^3}{Q^2K2}}$$

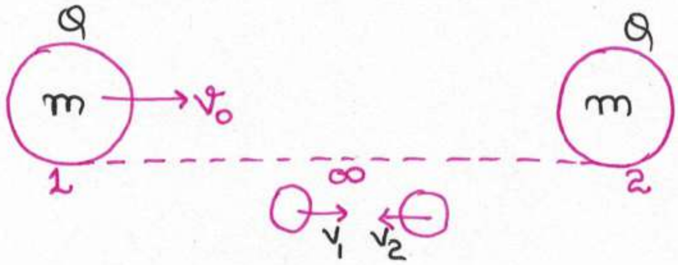
by energy method,

$$mv^2 - \frac{KQ^2}{\sqrt{R^2+4x^2}} = \text{Const.}$$

differentiate it.

Que.) Two particles are released in space & has velocity v_0 as shown.

(a) What is the min. Separation b/w the particles during this motion.



COM

$$\cancel{m}v_0 = \cancel{m}v_1 - \cancel{m}v_2$$
$$v_0 = v_1 - v_2$$

but $v_1 = -v_2$ (for min separation)

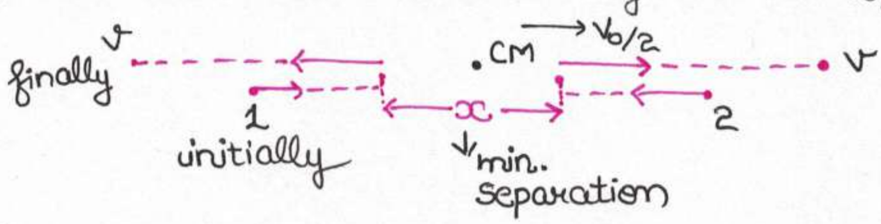
$$\therefore\ v_1 = \frac{v_0}{2} = v_2$$

COME

$$\frac{1}{2}mv_0^2 = +\frac{KQ^2}{x} + \frac{1}{2}mv_1^2$$
$$\frac{mv_0^2}{2} - \frac{mv_0^2}{4} = \frac{KQ^2}{x}$$
$$x = \frac{4KQ^2}{mv_0^2}$$

(b) Find the velocities of 1 and 2.

as the momentum is towards right the centre of mass will continue to move towards right with $v_0/2$.



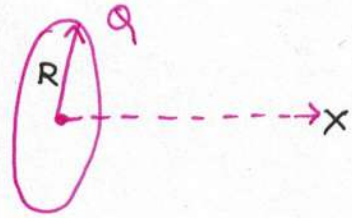
& by COM

$$v = \frac{v_0}{2}$$
$$\therefore\ v_1 = -v + \frac{v_0}{2} = 0$$
$$v_2 = v + \frac{v_0}{2} = v_0$$

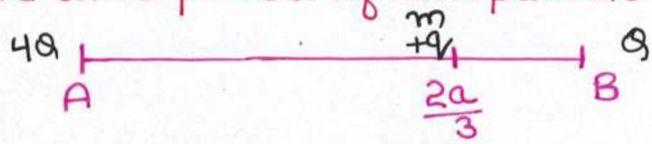
Que.) Find $\int_{x=\infty}^{x=0} E\,dx$. Its a ring

$$E\,dx = -dv$$

$$\int_{V_\infty}^{V_0} -dv = -V_0 + V_\infty$$
$$= -V_0$$



Que.) Find the time period of the particle.



$$F = \frac{4Qq}{\left(\frac{2a}{3}+x\right)^2} - \frac{Qq}{\left(\frac{2a}{3}-x\right)^2}$$

$$F = 4QqK\left(\frac{2a}{3}+x\right)^{-2} - Qq\left(\frac{a}{3}-x\right)^{-2}$$

$$F = \frac{9qKq}{a^2}\left[\left(1-\frac{3}{2}x\right) - \left(1-\frac{3x}{a}\right)^{-2}\right]$$

$$F = \frac{KQq\times q}{a^2}\left[\frac{9x}{a}\right]$$

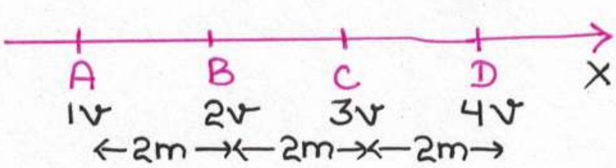
$$\therefore T = 2\pi\sqrt{\frac{ma^3}{81KQq}}$$

EQUIPOTENTIAL SURFACE

Def: These surfaces which have all points at the same potential.

★ There is no electric field tangential to equipotential surface. It means any possible field can be normal at it.

Que.)



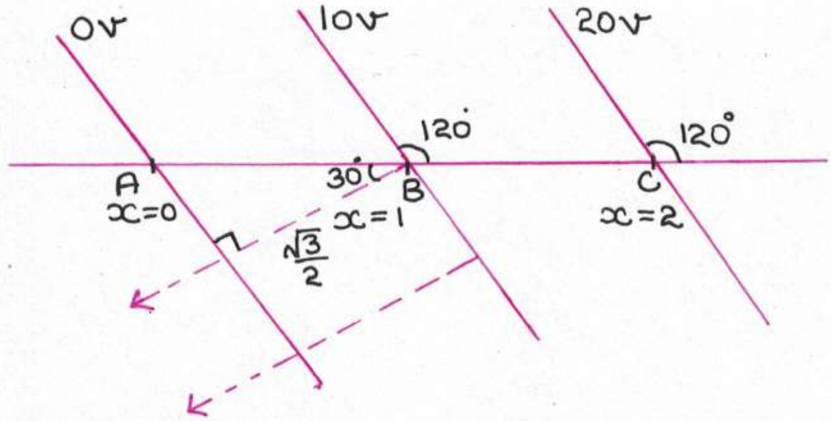
$$\frac{\delta v}{\delta x} = \frac{1}{2} = \text{Const.}$$

$$E_x = -\frac{1}{2}\, v/m$$

$$E = \sqrt{E_x^2 + E_y^2 + E_z^2}$$

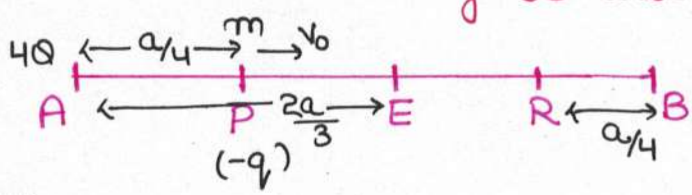
as we dont know about E_y & E_z,
therefore $E \geq \frac{1}{2}\frac{v}{m}$

Que.)



$$E = \frac{10}{\sqrt{3}} \times 2 \quad = \frac{20}{\sqrt{3}} = \text{gradient}$$

Que.) Find minimum velocity so that the particle reaches R.



$$\frac{1}{2}mv^2 - \frac{4Qq K \times 4}{a} - \frac{Qq \times 4K_2}{3a} - \frac{4Qq \times 3K}{2a} - \frac{Qq \times 3K}{a}$$

$$\frac{1}{2}mv_0^2 = \frac{52}{3}\frac{QqK}{a} - \frac{18QqK}{2a}$$

$$v_0^2 = \frac{(104Qq - 54Qq)K}{6am}$$

$$v_0^2 = \frac{\cancel{2} \times 50 QqK}{\cancel{6}_3 a}$$

$$v_0 = \sqrt{\frac{50QqK}{3am}}$$

ELECTRIC FIELD LINES (LINES OF FORCES)

- These are the imaginary lines, the tangent to which gives the direction of electric field.
- They emerges from positive charge/infinity and terminates at negative charge or infinity
- No. of lines emerging from point charge is proportional to point charge.
  i.e. n lines emerge from q.
  then 2n lines emerge from 2q.

- Lines of forces cannot intersect as every point has unique electric field.

SOLID ANGLE

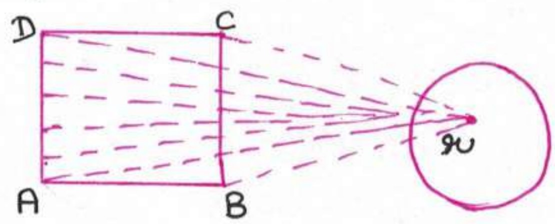
The angle subtended by a surface at a point in a space is called solid angle.

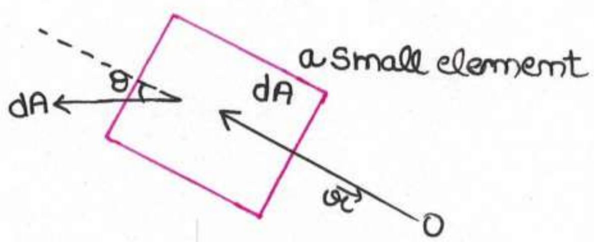
(Every close surface subtends 4π steradian at inside point)

The physical sense of solid angle is created by joining the boundary of surface to the concerned point.

$$\text{Solid angle in steradian} = \frac{\text{Area intercepted from the Sphere}}{(\text{Radius})^2}$$

taking centre at concerned point.

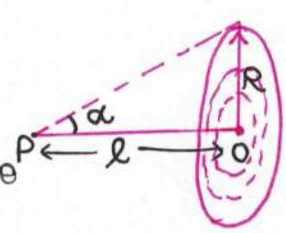




$\vec{dA}$ is $\perp$ to the surface

$$(d\omega)\text{Solid angle} = \frac{dA\cos\theta}{r^2}$$

Que.) Find solid angle subtended by disk at point P.



$$d\omega = \frac{2\pi x . dx \cos\theta}{(l \sec\theta)^2}$$

$$d\omega = \frac{2\pi \, l \tan\theta \, l \sec^2\theta . d\theta . \cos\theta}{l^2 \sec^2\theta}$$

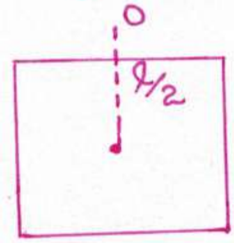
$$\left(\frac{x}{l} = \tan\theta \qquad \frac{1}{l} = \sec^2\theta \frac{d\theta}{dx} \right)$$

$$d\omega = 2\int_0^{\alpha} 2\pi \sin\theta . d\theta$$

$$d\omega = 2\pi(1-\cos\alpha)$$

SOLID ANGLE USING SYMMETRY

* Solid angle for square at point O.



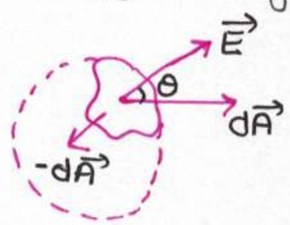
Imagine a cube and O will be its center.

$\therefore$ Solid angle $= \frac{4\pi}{6}$

ELECTRIC FLUX

No. of lines passing through an area is given by electric flux.

$d\vec{A}$ is outward of the volume of which dA is a part of



$$d\phi = \vec{E}\cdot d\vec{A}$$

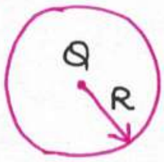
$$\phi = \int \vec{E}\cdot d\vec{A}$$

$$\phi = \oint \vec{E}\cdot d\vec{A} \quad \text{(for complete surface)}$$

$\phi = \vec{E}\cdot\vec{A}$ for a plane surface in uniform electric field.

Que.) Find flux over this sphere.

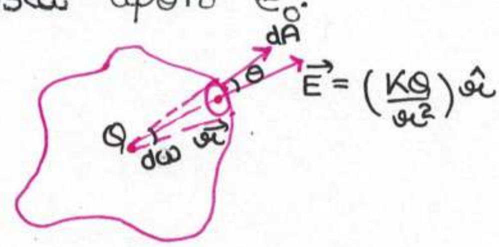
$$\phi = \frac{KQ}{R^2} \times 4\pi r^2$$



$$\phi = \frac{Q}{\epsilon_0}$$

GAUSS LAW

Net electric flux over a closed surface is equal to charge enclosed upon ϵ_0.



$$\vec{E}\cdot d\vec{A} = E\,dA\cos\theta$$

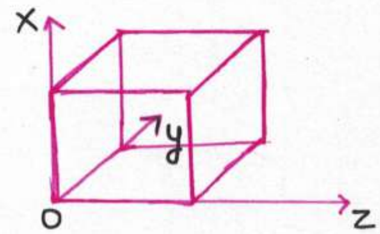
$$= KQ\left(\frac{dA\cos\theta}{r^2}\right)$$

$$\oint \vec{E}\cdot d\vec{A} = \int KQ \cdot d\omega$$

$$\oint \vec{E}\cdot d\vec{A} = KQ(4\pi)$$

$$\boxed{\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0} \quad \Bigg| \quad = \frac{Q}{\epsilon_0}\left(\frac{\omega}{4\pi}\right)}$$

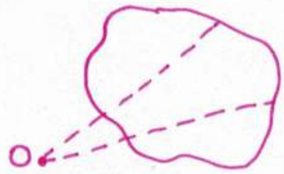
Que) Find the charge present inside the cube if
$\vec{E} = \left(1+\frac{x}{\ell}\right)\hat{i} + \alpha\hat{j} + p\hat{k}$



$$q_{in} = \epsilon_0 (2\ell^2 - \ell^2) = 0$$

$$q_{in} = \epsilon_0 \ell^2$$

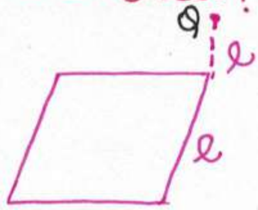
★ Electric flux through a closed surface in uniform electric field is always 0.



As solid angle subtended is same and opposite. The sum will be equal to zero.

Que.) Find the flux through this sheet?

$$\frac{Q}{\epsilon_0} \times \frac{1}{24} = \phi$$

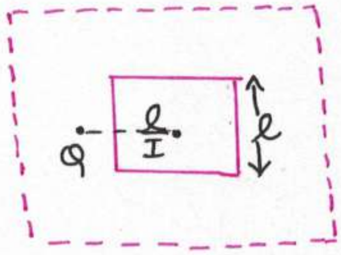


Que.) Find the flux for the ∞ sheet in which a square is cut out of length l.

$$\frac{Q}{4\pi\epsilon_0}(\omega_\infty - \omega_{squ.})$$

$$= \frac{Q}{4\pi\epsilon_0}\left(2\pi - \frac{4\pi}{6}\right)$$

$$= \frac{2Q}{3\pi\epsilon_0}$$



Que.) Find the flux through the curved surface.

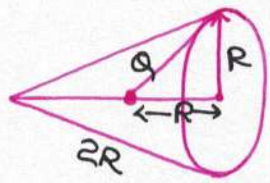
$$\omega = 2\pi(1-\cos\alpha)$$

$$\omega = 2\pi\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\omega_{CSA} = 4\pi - 2\pi - \frac{2\pi}{\sqrt{2}}$$

$$= 2\pi - \frac{2\pi}{\sqrt{2}}$$

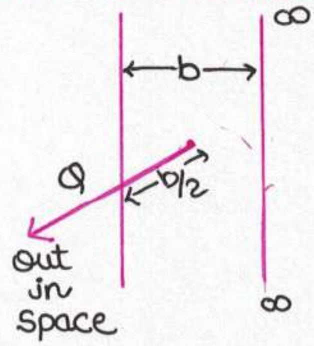
$$\phi = \frac{Q}{4\pi\epsilon_0}\left(2\pi - \frac{2\pi}{\sqrt{2}}\right)$$



Que.) Find the flux due to the ∞ sheet where width is b.

$$\phi = \frac{Q}{\epsilon_0}\left(\frac{\pi}{4\pi}\right)$$

$$\phi = \frac{Q}{4\epsilon_0}$$



Think it as cuboid of ∞ sheets without two closed end surface. Then each sheet will have a solid angle of π ie $\frac{4\pi}{4}$.

Que.) Find the flux through ABCD

For $x \to 0$

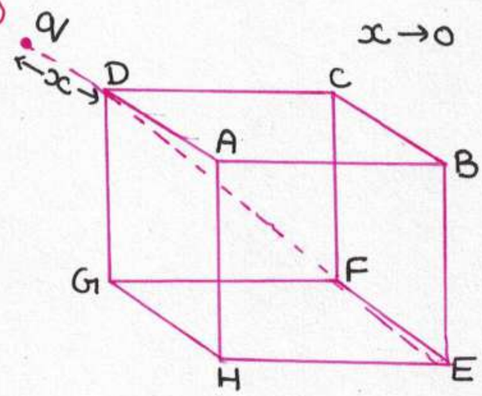
$$\overbrace{\phi_{ABCD} + \phi_{DAGH} + \phi_{DCFG}}^{\text{similar}} +$$

$$\overbrace{\phi_{ABEH} + \phi_{CBEF} + \phi_{GFEH}}^{\text{similar}} = 0$$

$$\downarrow q/24$$

$$\therefore 3\phi + \frac{3q}{24} = 0$$

$$\therefore \phi = \frac{q}{24}$$



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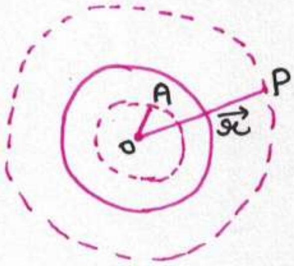
FIELD & POTENTIAL DUE TO SHELLS AND SPHERES

1. UNIFORMLY CHARGED SHELL



(i) By Gauss Law,

$$4\pi r^2 \times E = \frac{Q}{\epsilon_0}$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{KQ}{r^2}\hat{r}$$

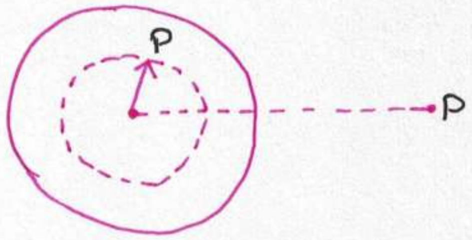
$$V = \frac{KQ}{r} \quad \left(\text{by integration as done earlier}\right)$$

(ii) For inside point A,

there is no charge, therefore no $\vec{E}$

$$\vec{E} = 0 \text{ and } V = \frac{KQ}{R} \quad (\text{as on the surface})$$

2. UNIFORMLY CHARGED SPHERE (SOLID)



(i) Similarly, for outside point P, by gauss law, we can say

$$\vec{E} = \frac{KQ}{r^2}\hat{r} \text{ and } V = \frac{KQ}{r}$$

(ii) For inside point P,

by gauss law,

$$E \times 4\pi r^2 = \frac{\sigma\left(\frac{4}{3}\pi r^3\right)}{\epsilon_0} \quad \left(\sigma \text{ is charge per unit volume}\right)$$

$$\left(\sigma = \frac{Q}{\frac{4}{3}\pi R^3}\right)$$

$$E = \frac{Qr}{4\pi R^3 \epsilon_0}$$

$$\boxed{E = \frac{KQr}{R^3}} \text{ and } \vec{E} = \frac{KQ\vec{r}}{R^3}$$

$$\int_{V_S}^{V} dV = -\int_{R}^{r} E.dr$$

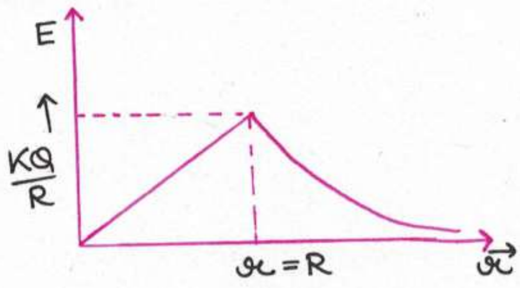
$$V - V_S = -\frac{KQ}{R^3}\int_{R}^{r} r.dr$$

$$V - V_S = \frac{-KQ}{R^3}\left(\frac{r^2 - R^2}{2}\right)$$

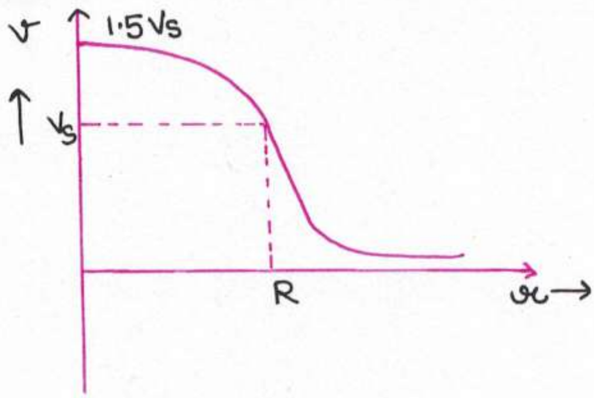
$$V = \frac{KQ}{R}\left[1 + \frac{R^2 - r^2}{2R^2}\right]$$

$$\boxed{V = V_S\left[1.5 - 0.5\left(\frac{r^2}{R}\right)\right]}$$

GRAPH FOR $\vec{E}$



GRAPH FOR V



For $r > R$, we have V inversely proportional to r.

For $r < R$, $\frac{dV}{dr} = \frac{-KQ}{R^3} r$

As r is increasing, magnitude of slope is increasing but remains -ve.

SPHERE & SHELL ANALYSIS - CONCLUSIONS

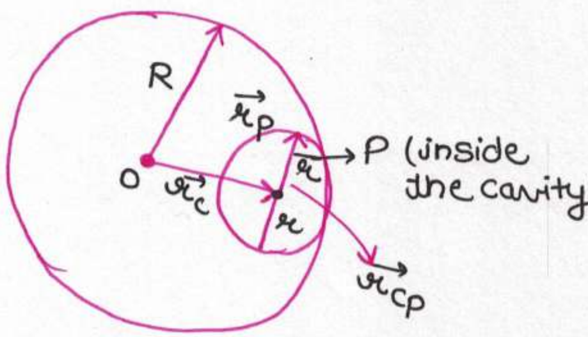
$$\vec{E}_{shell} = \begin{cases} 0 & r < R \\ \frac{KQ}{r^2} & r \geq R \end{cases}$$

$$V_{shell} = \begin{cases} \frac{KQ}{R} & r \leq R \\ \frac{KQ}{r} & r > R \end{cases}$$

$$\vec{E}_{sphere} = \begin{cases} \dfrac{KQ}{r^2} & r \geq R \\ \dfrac{KQr}{R^2} & r < R \end{cases}$$

$$V_{sphere} = \begin{cases} \dfrac{KQ}{r} & r \geq R \\ \dfrac{KQ}{r}\left[1.5 - 0.5\left(\dfrac{r}{R}\right)^2\right] & r < R \end{cases}$$

Que.) Find the $\vec{E}$ inside a cavity (spherical) inside a sphere of radius R.



$$\vec{E}_1 = \frac{KQ}{R^3}\vec{r}_p$$

$$\vec{E}_2 = \frac{KQ'}{r^3}\vec{r}_{cp}$$

$$\left(\frac{Q}{R^3} = \frac{Q'}{r^3}\right)$$

$$\vec{E} = \vec{E}_1 - \vec{E}_2$$

$$\vec{E} = \frac{KQ}{R^3}(\vec{r}_p - \vec{r}_{cp})$$

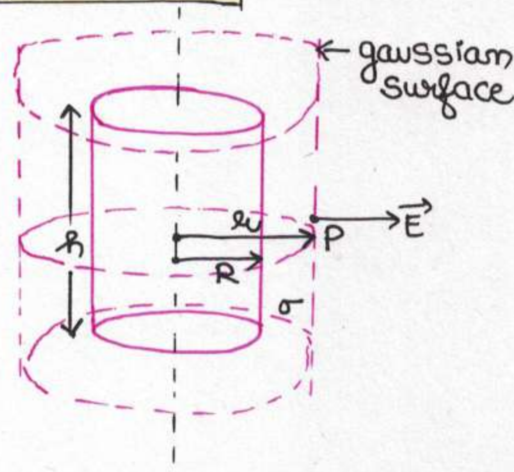
$$\vec{E} = \frac{KQ}{R^3}(\vec{r}_c)$$

$$\left(\frac{Q}{\frac{4}{3}\pi R^3} = \rho \Rightarrow \frac{Q}{R^3} = \rho \times \frac{4\pi}{3}\right)$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \times \frac{4\pi}{3}\rho(\vec{r}_c)$$

$$\vec{E} = \frac{\rho}{3\epsilon_0}(\vec{r}_c)$$

ELECTRIC FIELD DUE TO HOLLOW CYLINDER USING GAUSS LAW



For outside point,

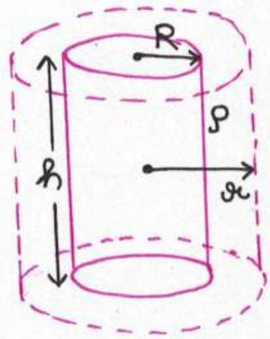
$$E \times 2\pi r h = \frac{2\pi R h \sigma}{\epsilon_0}$$

$$E = \left(\frac{2\pi R\sigma}{2\pi\epsilon_0}\right)\frac{1}{r}$$

★Notice it is same as $\frac{2K\lambda}{r}$ where λ is charge per unit length $= 2\pi R\sigma$

For inside Point $E = 0$

ELECTRIC FIELD DUE TO SOLID CYLINDER USING GAUSS LAW



$r \geq R$

$$E \times 2\pi r h = \frac{\rho \pi R^2 h}{\epsilon_0}$$

$$E = \frac{\rho R^2}{2 r \epsilon_0} = 2K\rho r$$

$$E = \frac{2K}{r}(\rho r^2)\pi$$

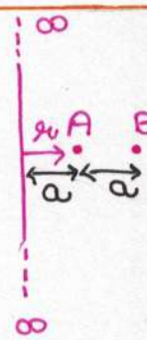
$$\therefore \lambda = \rho r^2 \pi \quad \text{(charge / length)}$$

$r < R$

$$2\pi r h \times E = \frac{\rho(\pi r^2)h}{\epsilon_0}$$

$$E = \frac{\rho r}{2\epsilon_0}$$

NOTE: Here we cannot talk about potential at A & B because for ∞ wire we cannot say potential at ∞ to be 0. We can't relate ∞ wire & ∞ distance but we can talk about potential difference.



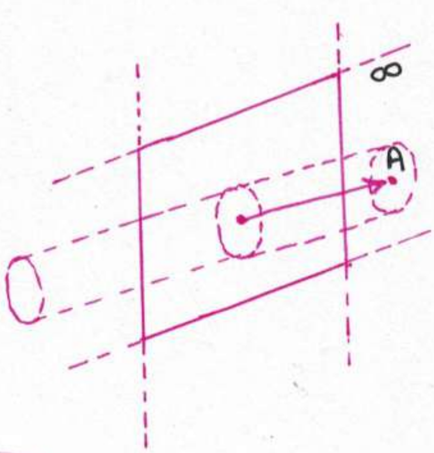
$$V_B - V_A = dV$$

$$= -\int \frac{2K\lambda}{r} \cdot dr$$

$$= -2K\lambda \ln r \Big]_a^{a+a}$$

$$dV = -2K\lambda(\ln 2)$$

ELECTRIC FIELD DUE TO ∞ SHEET

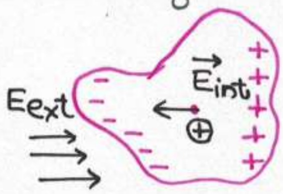


$$E(2\pi r^2) = \frac{\sigma(\pi r^2)}{\epsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

PROPERTIES OF CONDUCTOR

1.) Under electrostatic conditions, electric field inside the volume of conductor is zero.



$$E_{ext} - E_{int} = 0 = E_{net}$$

then electrostatic condition

$$E_{ext} - E_{int} = E_{net} \neq 0$$

then transient condition.

2.) Under electrostatic condition, excess charge resides on the surface of conductor.

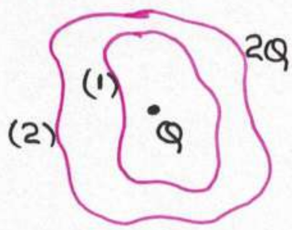
Since $\vec{E} = 0$,

$q_{in} = 0$

Que.) Find the charge on (1) & (2).

(1) $\rightarrow Q$

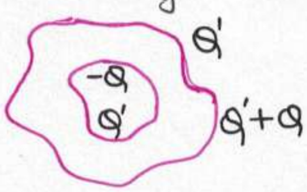
(2) $\rightarrow 3Q$



3.) Since $E = 0$, because surface is inside conductor,

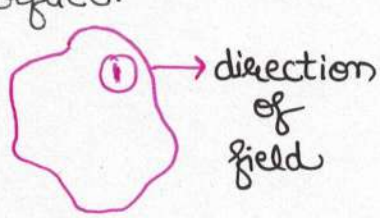
$\therefore q_{in} = 0$

Hence charge equal and opposite is induced at the inner surface of cavity. Net charge remains same.



4.) Hence all points on a conductor are at same potential which is equal to potential on the surface. Hence it is an equipotential surface.

5.) At point (1), (Since it is equipotential), field component cannot be along tangential direction. Hence it has to be normal to the surface.



6.) When two conductors are joined through a conducting wire, they share the charges among them selves, till the potential become same.

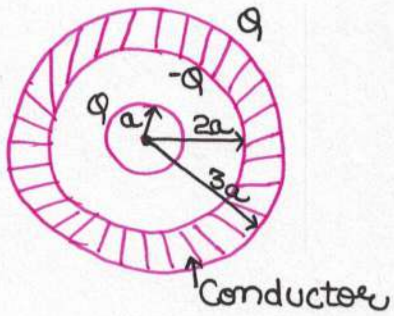
EARTHING

It means connecting to earth. Conductor share charge with earth, till its potential become 0.

$$V_{earth} = 0 \quad \text{(assumed)}$$

Que.) Find the potential at the last surface.

$$= \frac{KQ}{3a}$$



Que.) Find the potential at the inside shell.

$$\frac{KQ}{a} - \frac{KQ}{2a} + \frac{KQ}{3a} = \frac{5}{6}\left(\frac{KQ}{a}\right)$$

Que.) Find the charge flown to earth.

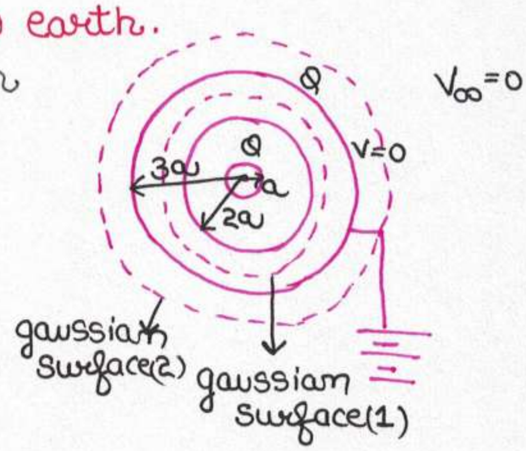
As V field at both gaussian surface are zero, the charge inside is zero

$\therefore E = 0$

as $dV = 0$

Thus Q charge has flown.

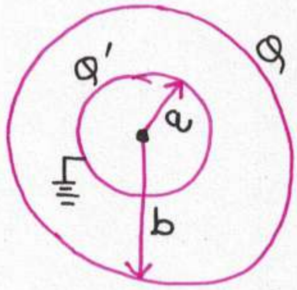
We can also find it by making potential 0.



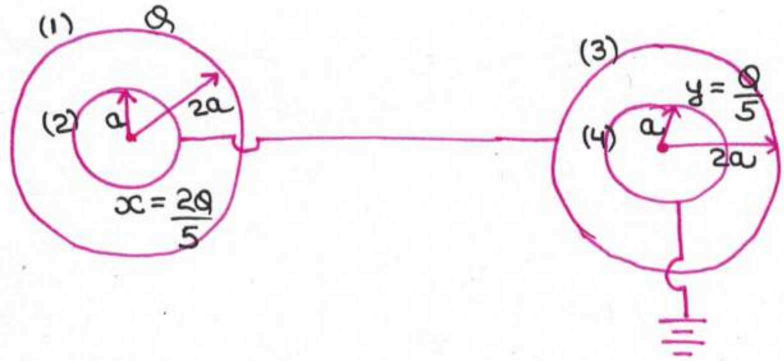
Que.) Find the charge on surface of the inner sphere.

$$\frac{KQ'}{b} + \frac{KQ}{a} = 0$$

$$Q' = -\frac{Qa}{b}$$



Que.)



As charge (2) & (3) is zero, initially even if the charge flows to one, the total charge remains zero.

$$\frac{KQ}{2a} + \frac{Kx}{a} = K\left(\frac{-x}{2a} + \frac{y}{2a}\right)$$

$$\frac{Q}{2} + x = \frac{-x}{2} + \frac{y}{2}$$

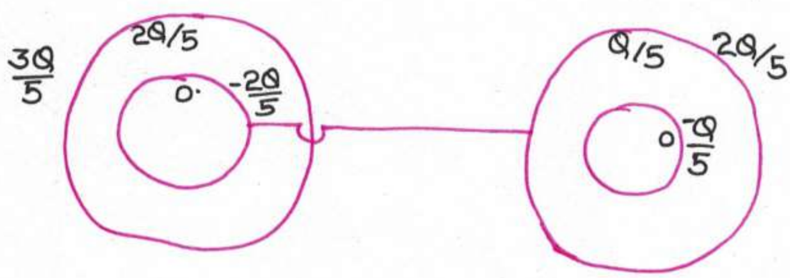
$$\frac{-Kx}{2a} + \frac{Ky}{a} = 0$$

$$\boxed{y = \frac{x}{2}}$$

$$Q + 3x = \frac{x}{2}$$

$$2Q + 6x = x$$

$$x = -\frac{2Q}{5}$$



Que.) What will be the surface charge distribution in previous question.

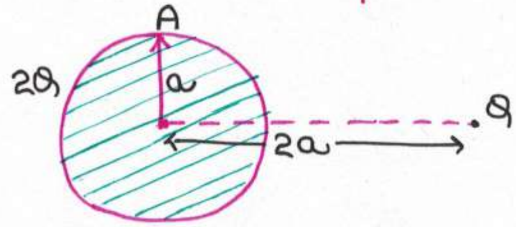
$1 \longrightarrow \frac{3Q}{5}$ $\qquad$ $1' \longrightarrow \frac{2Q}{5}$

$2 \longrightarrow -\frac{2Q}{5}$ $\qquad$ $2' \longrightarrow 0$

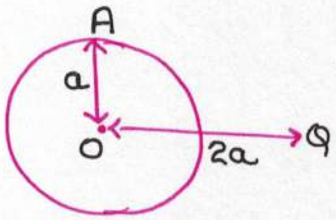
$3 \longrightarrow Q/5$ $\qquad$ $3' \longrightarrow Q/5$

$4 \longrightarrow -Q/5$ $\qquad$ $4' \longrightarrow 0$

Que.) Find potential at A. The sphere is a conductor.



When Q is brought near the sphere, the 2Q charge on the sphere is not uniformly distributed.



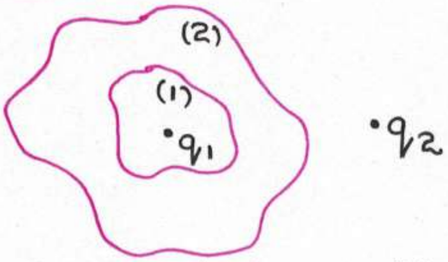
$$V_0 = \underbrace{\frac{KQ}{2a}}_{\text{due to point charge}} + \underbrace{\frac{K(2Q)}{a}}_{\text{due to charge on the sphere}}$$ (potential at the center)

As it is a conductor the inside field is zero, thats why

$$V_0 = V_A$$

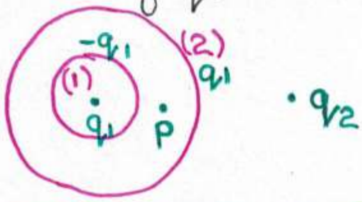
$$V_A = \frac{5KQ}{2a}$$

NOTE:



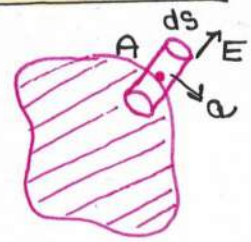
If we consider a conductor with q_1 charge inside the cavity and the conductor is neutral as a whole. Thus $-q_1$ charge is induced on surface (1) and $+q_1$ is induced on (2). If we move the charge q_1 inside the cavity then the charge on surface (1) will redistribute itself but not the charge on surface (2). Similarly if we move q_2 then only charge on surface (2) will redistribute itself.

When q_2 is moved then redistribution of q_1 on (2) makes the field zero at point P but that doest not need the redistribution of q_1 on surface (1).



ELECTRIC FIELD NEAR A CONDUCTOR'S SURFACE

$$E \cdot ds = \frac{\sigma\, ds}{\epsilon_0}$$



(E is ⊥ to the surface)

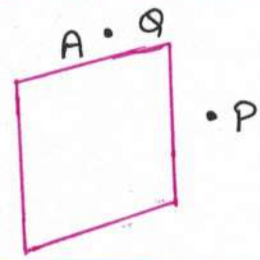
$$\boxed{E = \frac{\sigma}{\epsilon_0}}$$

(σ is the charge density of surface facing the point)

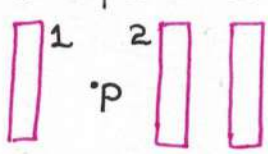
Que.) If a long conducting sheet is taken with charge Q and area A. Find electric field at a point near its surface.

$$\sigma = \frac{Q}{2A}$$

$$E_P = \frac{\sigma}{\epsilon_0} = \frac{Q}{2A\epsilon_0}$$



NOTE: To find electric field at point P, use formula $\frac{\sigma}{\epsilon_0}$ where σ is the charge density of either of the surfaces facing the point. In this case surface 1 & 2.

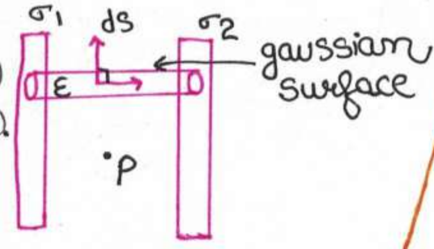


When we had found the electric field near a conducting surface, we have used gauss law and in gauss law we talk about the net enclosed charge and the net electric field.

Thus, when two long conducting surfaces are placed parallel to each other, at some finite distance then electric field due to any one of them will be same and should give the same result & that will be the net electric field.

$\phi = 0$ (as $E = 0$ inside conductor)

∴ Total inside charge should be equal to 0.



$$\sigma_1 + \sigma_2 = 0$$

$$E_P = \frac{\sigma_1}{\epsilon_0} = \frac{-\sigma_2}{\epsilon_0}$$

.... $\therefore$ $\boxed{\sigma_1 = -\sigma_2}$

which we can see through the gauss law also.

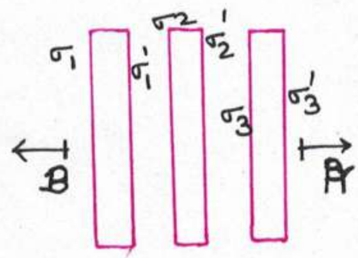
Here σ_1 & σ_2 are final charge densities.

NOTE: Some results

1. $\sigma_1' = -\sigma_2$
2. $\sigma_2' = -\sigma_3$
3. $\sigma_1 = \sigma_3'$

(as $E_B = E_A$

$$\frac{\sigma_1}{\epsilon_0} = \frac{\sigma_3'}{\epsilon_0}$$

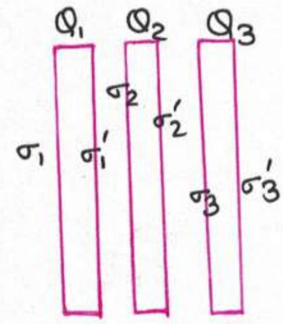


$E_B = E_A$ because due to each sheet the field is independent of distance.

An important Result:

$$\sigma_1 A + \sigma_3' A = Q_1 + Q_2 + Q_3$$

$$\boxed{\sigma_1 = \sigma_3' = \frac{Q_1 + Q_2 + Q_3}{2A}}$$



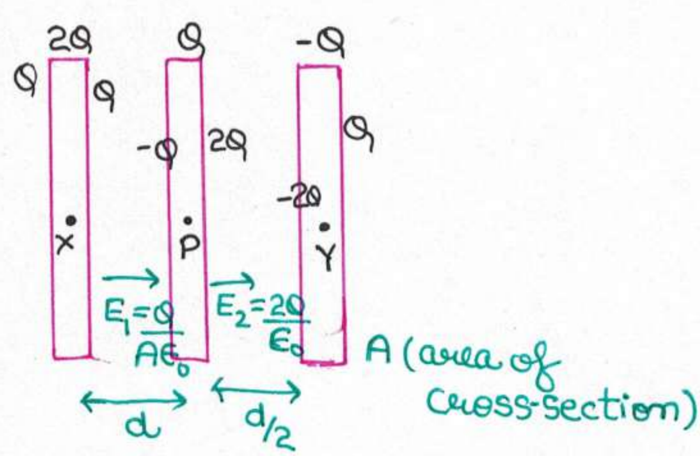
Que.) Find $V_Y - V_X$

$$V_P - V_x = -E_1 d$$

$$V_y - V_P = -E_2 \frac{d}{2}$$

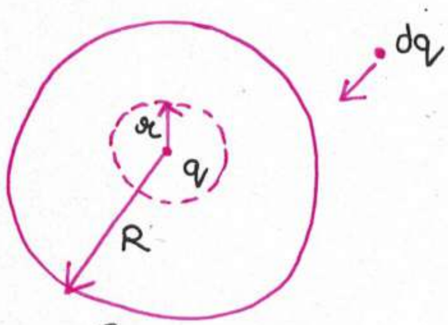
$$V_y - V_x = -\left(E_1 d + \frac{E_2 d}{2}\right)$$

$$= -\left(\frac{2Qd}{A\epsilon_0}\right)$$



SELF ENERGY OF A SOLID SPHERE

$$W = \int \frac{Kq}{r} \cdot dq$$



$$W = \int_0^Q \frac{Kq}{(q/Q)^{1/3} R} \cdot dq$$

$$\frac{Q}{\frac{4}{3}\pi R^3} = \frac{q}{\frac{4}{3}\pi r^3}$$

$$r = \left(\frac{q}{Q}\right)^{1/3} R$$

$$W = \int_0^Q \frac{Kq^{2/3} \cdot Q^{1/3}}{R} \cdot dq$$

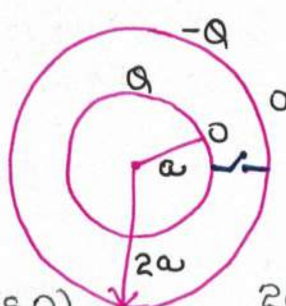
$$W = \frac{3}{5} \frac{KQ^{5/3}}{R} \cdot Q^{1/3}$$

$$\boxed{W = \frac{3}{5} \frac{KQ^2}{R}}$$

NOTE: Self energy should be considered when the size of the sphere is changing or the total charges changes with same distribution & we are applying conservation of energy. But, in case the size remains same and only the charge distribution changes then no need to consider self energy.

Que.) What is the heat dissipated when the switch is closed?

Here the final charges are written by after switching on.



$$\frac{\cancel{K}(Q-q)}{\cancel{a}} = \frac{\cancel{K}(Q}{2a}$$

$$2Q - 2q = Q - q$$
$$(Q = q)$$

Heat dissipated = Initial energy − Final energy

$\because$ Final energy = 0 (as final charge is 0)

$\therefore$ Heat dissipated = Self energy$_{initial}$ + Interaction energy$_{initial}$

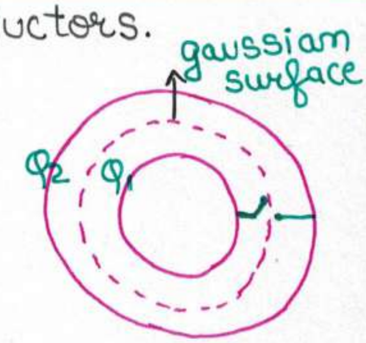
$$\text{Heat dissipated} = K\left(\frac{Q^2}{2a} + \frac{Q^2}{4a} - \frac{Q^2}{2a}\right)$$

$$= \frac{KQ^2}{4a}$$

NOTE: The spheres shown are conductors. when the switch is switched on, shphere 1 & sphere 2 will be at same potential.

$\therefore dV = 0 \Rightarrow E = 0$

$\therefore$ Electric field between the two spheres is zero.

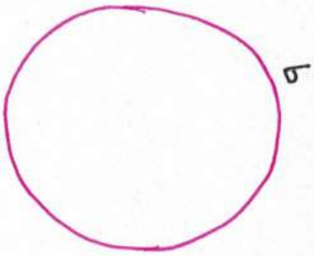


If we consider the gaussian surface as shown, $\phi = 0$, therefore $q_{IN} = 0$

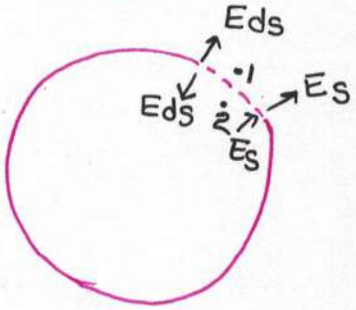
Imp: It means the charge on the inside sphere has to be zero.

ELECTROSTATIC PRESSURE

Consider a shell with σ as surface charge density.



Remove a small portion of shell say dS and say the left portion is S. Consider two points (1) and (2) very near to the surface S, inside and outside.



Now the electric field due to dS on point 1 and point 2 would be same in magnitude, say E_{ds}.

But for the remaining portion S, point 1 and point 2 are almost same. Thus, electric field due to S on 1 and 2 would be same, say E_S.

Net electric field on point 1 = Electric field due to the whole sphere.

$$\therefore E_{ds} + E_S = E$$

As (2) is inside point and electric field inside the shell is zero,

$$E_{ds} - E_S = 0$$

$$\boxed{\therefore E_{ds} = \frac{E}{2}}$$

NOTE 1: Electric field in any shell or conductor due to a small portion ds would be half of the electric field due to the whole conductor, but only for nearby points.

$$\text{Electrostatic pressure} = \frac{\text{Force exerted by (Total surface} - ds)}{\text{Small area } (ds)}$$

$$= \frac{E/2\,\sigma\, ds}{ds}$$

$$\boxed{\text{Electrostatic Pressure} = \frac{E}{2}\sigma}$$

Here E is the electric field due to whole body.

NOTE 2: The same result can be used for conductors. It is because in conductors, also, the electric field inside is zero as in shell.

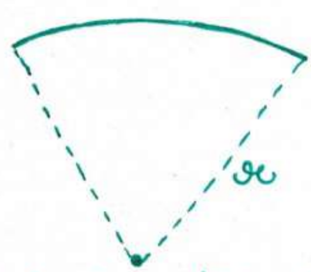
$$\boxed{\text{Electrostatic Pressure of a Conductor} = \frac{\sigma^2}{2\epsilon_0}}$$

Important: In any arbitrary shaped conductor,

(i) Surface charge density is more where the curvature is more and the radius of curvature is less.

(ii) Surface charge density is less where the curvature is less and the radius of curvature is more.

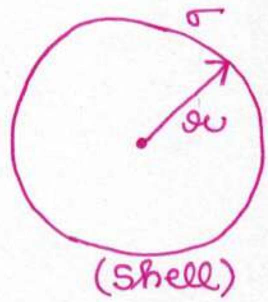
r↓, Curvature ↑



r↑, Curvature ↓

We can give the proof for spherical shell and can assume that it is true for an arbitrary shaped conductor to some extent by saying that potential at the conductors surface is governed by the inside portion near to it.

$$V_{\substack{at \\ surface}} = \frac{1\ \sigma\ 4\pi R^2}{(4\pi\epsilon_0)\times R}$$



$$V_{\substack{at \\ surface}} = \frac{\sigma R}{\epsilon_0}$$

as V is constant (σR) is constant

If $R\downarrow$, $\sigma\uparrow$ and if $R\uparrow$, $\sigma\downarrow$

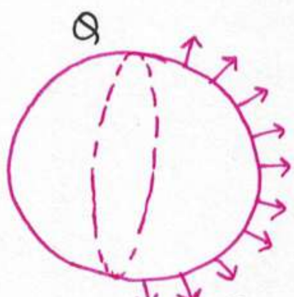
Que.) Radius of soap bubble of conductivity liquid is R, surface tension is S. What charge should be placed on the bubble, so that pressure inside is same as outside?

$$\frac{4T}{R} = \frac{\sigma^2}{2\epsilon_0}$$

$$\sigma = \sqrt{\frac{8T\epsilon_0}{R}}$$

$$Q = 4\pi R^2\sqrt{\frac{8T\epsilon_0}{R}}$$

Que.) Two hemispherical shells are just together having a total charge Q uniformly distributed. Find the force exerted by the one on the other.



El. pressure $\times$ area $= ds \times \frac{\sigma^2}{2\epsilon_0}$

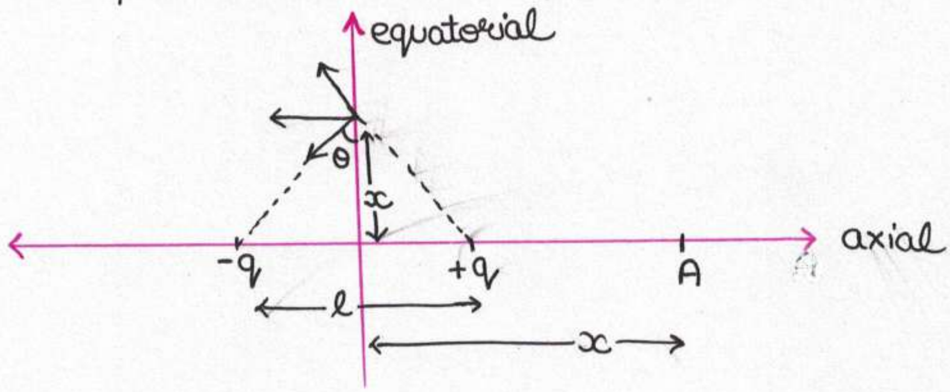
To find total force, we can use the projection area

$$\frac{Q^2}{2(4\pi R^2)^2\epsilon_0}\times \pi R^2 = F_{net}$$

$$\frac{Q^2}{32\pi R^2\epsilon_0} = F_{net}$$

DIPOLE

Dipole is a combination of 2 equal and opposite charges with some separation between them.



$$\text{Dipole } \vec{p} = (q\,l)\,\hat{n}$$ (direction is from -q to +q)

POINT DIPOLE

When point of consideration is too far from dipole, the distance between the charges is negligible and the dipole is called point dipole.

$$E_A = \frac{Kq}{(x-\frac{l}{2})^2} - \frac{Kq}{(x+\frac{l}{2})^2}$$

$$= Kq\left[\frac{(x+l/2)^2-(x-l/2)^2}{(x-\frac{l}{2})^2\,(x+\frac{l}{2})^2}\right]$$

$$= Kq\left[\frac{2xl}{x^4}\right]$$

$$= \frac{2K(ql)}{x^3}$$

$$= \frac{2K\vec{p}}{x^3}$$

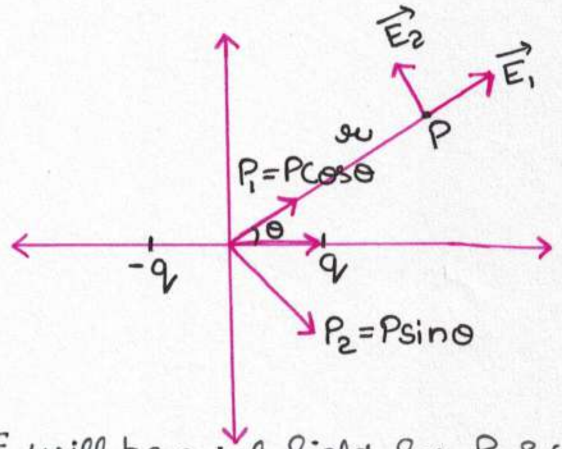
$$\boxed{\vec{E}_{axial} = \frac{2K\vec{p}}{x^3}}$$

$$E_E = \frac{2Kq}{\left[\left(\frac{l}{2}\right)^2+x^2\right]}\cdot\cos\theta$$

$$E_E = \frac{\cancel{2}Kq}{\left(\frac{l^2}{4}+x^2\right)^{3/2}} \times \frac{l}{\cancel{2}}$$

$$E_E = \frac{-K\vec{P}}{x^3}$$

$$\boxed{\vec{E}_{equatorial} = \frac{-K\vec{P}}{x^3}}$$



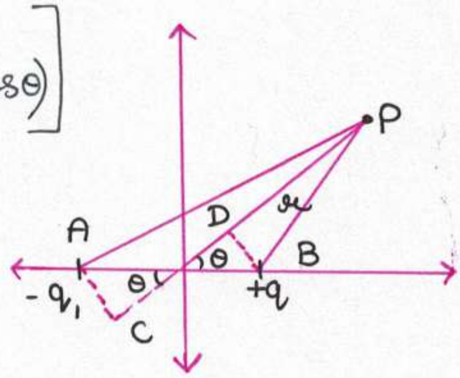
$$|E| = \sqrt{E_1^2+E_2^2} = \frac{KP}{r^2}\sqrt{1+3\cos^2\theta}$$

$$\tan\alpha = \frac{E_2}{E_1} = \frac{1}{2}(\tan\theta)$$

E_1 will be axial field for P_1 & E_2 will be equatorial field due to

POTENTIAL DUE TO DIPOLE

$$V = K\left[\frac{q}{\left(r-\frac{l}{2}\cos\theta\right)} - \frac{q}{\left(r+\frac{l}{2}\cos\theta\right)}\right]$$



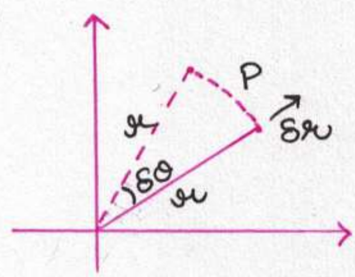
$$V = \frac{Kql\cos\theta}{r^2}$$

$$V = \frac{Kp\cos\theta}{r^2}$$

$AP \simeq CP$ and $BP \simeq DP$

$\therefore AP = r+\frac{l}{2}\cos\theta$ and $BP = r-\frac{l}{2}\cos\theta$

Now, we can see that



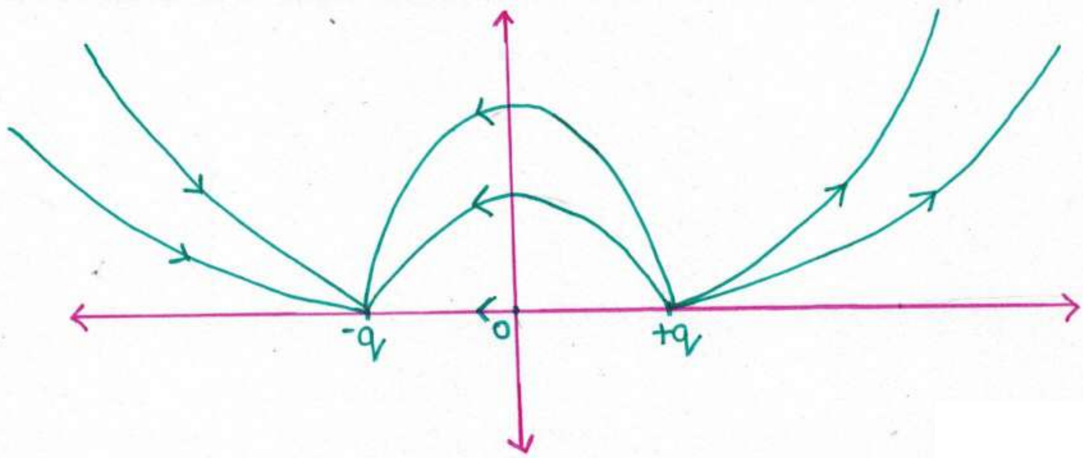
$$\vec{E_1} = -\frac{\delta V}{\delta r} = \frac{2K\vec{P}\cos\theta}{r^3}$$

(θ constant)

$$\vec{E_2} = -\frac{\delta V}{\delta r\theta} = -\frac{\delta V}{r\,\delta\theta} = \frac{-2K\vec{P}\sin\theta}{r^3}$$

(r constant)

ELECTRIC FIELD LINES DUE TO A DIPOLE



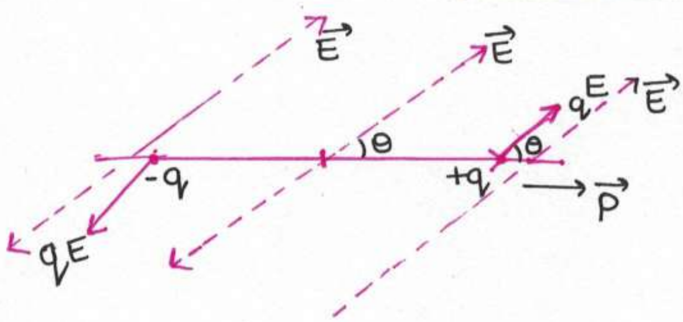
NET FORCE & BEHAVIOUR OF DIPOLE IN UNIFORM ELECTRIC FIELD

$$\vec{F} = q\vec{E_1} - q\vec{E_2}$$

$$\vec{F} = q(\vec{E_1} - \vec{E_2})$$

In uniform electric field $\vec{E_1} = \vec{E_2}$ and thus $\vec{F} = 0$.

TORQUE



Centre of mass of dipole is not at the centre, & ∵ due to couple, is independent of the center of mass.

$$\vec{\tau} = 2\left(qE\frac{l}{2}\sin\theta\right)$$

$$\vec{\tau} = qEl\sin\theta$$

$$\vec{\tau} = pE\sin\theta$$

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

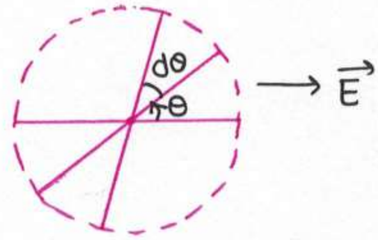
If θ is very small, then dipole will undergo S.H.M.

Que.) Find under assumption that $U = 0$ at $\theta = 90°$. Find the potential energy of dipole angle θ.

$$\int_{U}^{0} dU = \int_{\theta}^{\frac{\pi}{2}} pE\sin\theta . d\theta$$

$$-U = PE\cos\theta$$

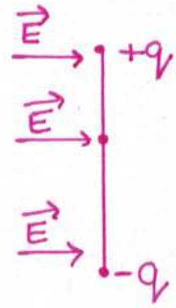
$$\boxed{U = -PE\cos\theta}$$



★ Why U is taken 0 at $\theta = \pi/2$.

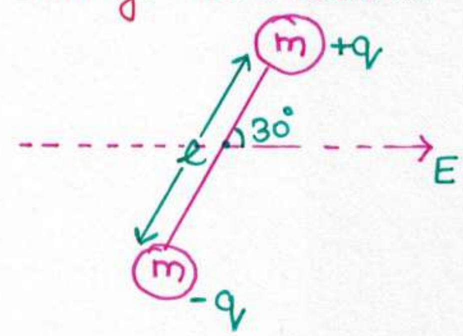
$$P.E._{pq} = -P_{E.-q}$$

$$\therefore \text{Total } P.E. = 0$$



Que.) A dipole is released as shown in uniform electric field. Calculate max. angular velocity of rod during its motion.

angular velocity is max. when P.E. is minimum.



$$\therefore \theta = 0°$$

$$PE = -PE$$

$$-PE\cos 30° = \frac{1}{2}I\omega^2 - PE\cos 0°$$

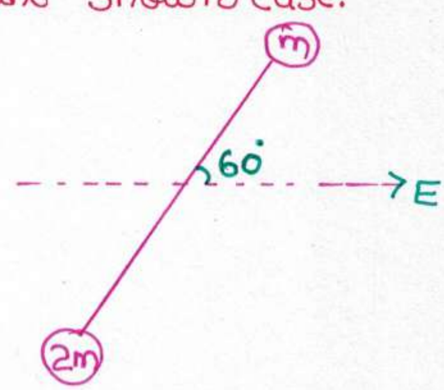
$$PE\left(1-\frac{\sqrt{3}}{2}\right) = \frac{1}{2}I\omega^2 \qquad \left(I = 2\times\frac{ml^2}{4}\right)$$

$$\omega = \sqrt{\frac{2qE(2-\sqrt{3})}{ml}}$$

Que.) Repeat the previous question for the shown case.

Here the $-F = 0$, $\therefore \vec{a}_{cm} = 0$

That means the rotation is fix axis rotation about centre of mass.



$$-PE\cos 60° = \frac{1}{2}I_{cm}\omega^2 - PE\cos 0°$$

$$\therefore \frac{PE}{2} = \frac{1}{2}I_{cm}\omega^2$$

$$\left(I_{cm} = (2m)\left(\frac{l}{3}\right)^2 + m\left(\frac{2l}{3}\right)^2\right)$$

$$\omega = \sqrt{\frac{PE}{I_{cm}}}$$

Que.) $\vec{E} = \left(\frac{c}{x^3}\right)\hat{i}$

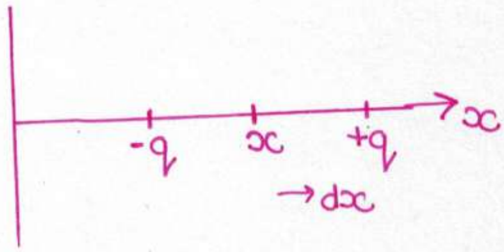
A dipole of $\vec{p}$ lies along x-axis. Calculate force on dipole. Dipole lies at $(x,0)$; ie the center of the dipole.

$$F = q(\vec{E_1} - \vec{E_2})$$

$$E_1 - E_2 = \left(\frac{dE}{dx}\right) l(\hat{i})$$

$$F = q\left(\frac{dE}{dx}\right) l$$

$$\vec{F} = \frac{3cp}{x^4}(-\hat{i})$$

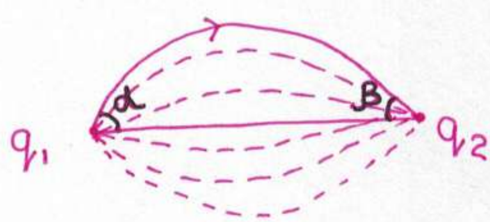


(change in electric field per unit length × length)

ELECTRIC FIELD LINES (LINES OF FORCES)

- These are the imaginary lines, the tangent to which gives the direction of electric field.
- Electric field lines can never intersect.
- They emerges from positive charge or infinity and terminates at negative charge or infinity.
- Electric field lines can never form a closed loop.

Que.) Electric field line is as shown. Relate α, β in terms of q_1 and q_2.



Field lines are proportional to charge and electric field lines permit Solid angle is also proportional to charge.

Solid angle for $q_1 = 2\pi(1-\cos\alpha)$

Solid angle for $q_2 = 2\pi(1-\cos\beta)$

let us say that N lines are emitted from q_1 inside the fig.

$$\frac{N}{2\pi(1-\cos\alpha)} \propto q_1$$

$$\frac{N}{2\pi(1-\cos\beta)} \propto q_2$$

The proportionality constant would be same

$$\therefore \boxed{\frac{2\pi(1-\cos\alpha)}{2\pi(1-\cos\beta)} = \frac{q_2}{q_1}}$$

Que.) Find the interaction force b/w the dipole.

$$\vec{E_B} = \frac{2KP_1}{r^3}$$

$$\vec{F_B} = \left[\left(\frac{\delta E_B}{\delta r}\right) \times l_2\right] q$$

$$= -\frac{6KP_1}{r^4}(l_2 q)$$

$$\vec{F_B} = \frac{-6KP_1P_2}{r^4}$$

Alternative:

We can find the potential energy of a dipole by considering a point dipole and find its potential energy at its center.

$$u = -\left(\frac{2KP_1}{r^3}\right)P_2 \qquad (u = -PE)$$

$$\vec{F_B} = -\frac{\delta u}{\delta r} = \frac{d}{dr}\left(\frac{2KP_1P_2}{r^3}\right)$$

$$= \frac{-6KP_1P_2}{r^4}$$

CENTRE OF MASS

It is used if forces are parallel on every point on mass of body. for eg. - Gravitational Force.

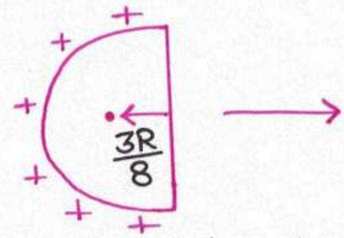
CENTRE OF CHARGE

It is used only if body is kept in uniform electric field.

NOTE: COM = CoC

when charge is distributed uniform throughout the volume.

Eg:-



at $\left(\frac{3R}{8}\right) \rightarrow$ CoC

Solid hemisphere